A.

Ch-3

Hyper Surfaces

① Σ eqn ① $\phi(x^\alpha) = 0$

② Parametric

$x^\alpha = x^\alpha(y^a)$

② Normal $n_\alpha \propto \partial_\alpha \phi$

for Non Null $\quad n_\alpha n^\alpha \equiv \epsilon \equiv \begin{cases} 1 & \Sigma \text{ spacelike} \\ -1 & \Sigma \text{ timelike} \end{cases}$

Normalized

Convention:- n^α point in the direction of increasing ϕ

$\therefore n^\alpha \partial_\alpha \phi > 0$??

③ From these 2 we can get that

$$n_\alpha = \frac{\epsilon \partial_\alpha \phi}{|g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi|^{1/2}}$$

④ This will not work for Null case

as $\partial_\mu \phi \partial^\mu \phi = 0$

$\therefore n_\alpha \longrightarrow \infty$

⑤ Hence normal can't be Normalized in Null Case

$\therefore$ let $k_\alpha = \partial_\alpha \phi$

when $\phi \uparrow$ in future k_α should be future directed

⑥ As $k^\alpha k_\alpha = 0 \quad \therefore k^\alpha$ is Tangent to Σ

(7) ~~Null Curves are always~~

Th. Given a H.S. Null, we are given Null Curves. These Null Curves always follow Null Geodesic.

Th. H.S. is generated by null geodesics & k^α is tangent to the generator.

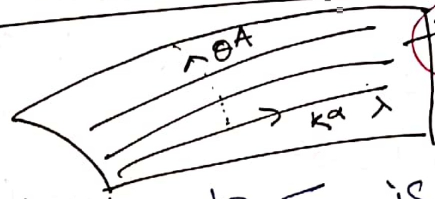
(8) **Th.** In general λ is non affine parameter

But

when $\phi = \text{const}$. describe whole family of null H.S. then $\lambda = $ affine.

(9) Coordinates on H.S. $\equiv$ Intrinsic Coordinates

Null case $y^a = (\lambda, \theta^A)$



$\rightarrow$ 3D H.S.

$\Rightarrow$ By const. $k_\alpha e^\alpha_A = 0$

why?

(10) The metric intrinsic to Σ is obtained by restricting line element to Σ.

Def: $e^\alpha_a \equiv \dfrac{\partial x^\alpha}{\partial y^a} \Rightarrow$ Tangent to Σ

$\therefore \; e^\alpha_a n_\alpha = 0$

(11) **Null case**

$e^\alpha_a \equiv \dfrac{\partial x^\alpha}{\partial y^a} \Rightarrow$ Tangent to Σ

$$e^\alpha_a = \left(\frac{\partial x^\alpha}{\partial y^1}, \frac{\partial x^\alpha}{\partial y^2}, \frac{\partial x^\alpha}{\partial y^3}\right) = \left(\frac{\partial x^\alpha}{\partial \lambda}, \frac{\partial x^\alpha}{\partial y^A}\right)$$

$$e^\alpha_a = \left(k^\alpha, \; e^\alpha_A\right)$$

$k_\alpha e^\alpha_A = \text{const}$

but

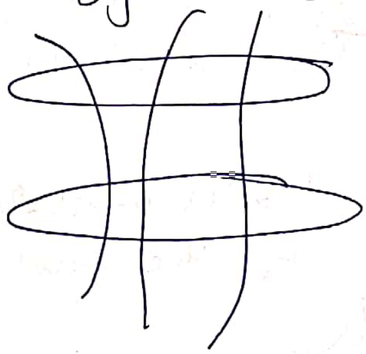
$k_\alpha e^\alpha_A = 0$

(12) **Non Null**

$$dx^\alpha = \frac{\partial x^\alpha}{\partial y^a} dy^a + \frac{\partial x^\alpha}{\partial \tau} d\tau$$

$$ds^2_\Sigma = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$dx^\alpha_\Sigma = \left(\frac{\partial x^\alpha}{\partial y^a}\right)_\tau dy^a$$



$\tau = \text{const}$

$$ds^2_\Sigma = \left(g_{\alpha\beta}\, e^\alpha_a e^\beta_b\right) dy^a dy^b$$

$$\equiv h_{ab}\, dy^a dy^b \Longrightarrow h_{ab} \text{ is Symmetric}$$

$$h_{ab} \equiv g_{\alpha\beta}\, e^\alpha_a e^\beta_b = \text{Induced metric}$$

(13) Th. → h_{ab} is scalar w.r.t. transf. of spacetime coord.
→ h_{ab} is Tensor w.r.t. transf. of H.S. coord.

$h_{ab} \equiv$ 3-Tensor.

$\boxed{\text{we can choose any coord. But lets take this}}$

(14) **Null Case**

H.S. $y^a = (\lambda, \theta^A) = 3D$

$$e^\alpha_1 = \frac{\partial x^\alpha}{\partial \lambda} = k^\alpha$$

$$h_{ab} \equiv g_{\alpha\beta}\, e^\alpha_a e^\beta_b$$

$$h_{11} = g_{\alpha\beta}\, k^\alpha k^\beta = 0$$

$$h_{1A} = g_{\alpha\beta}\, k^\alpha e^\beta_A = k^\alpha e_{\alpha A} = 0 \quad (\text{By Construction})$$

$$h_{ab} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h_{22} & h_{23} \\ 0 & h_{32} & h_{33} \end{pmatrix} = \text{Symetric} \quad \therefore \underline{h_{23} = h_{32}}$$

$$h_{32} = h_{23}$$

$$h = |h_{ab}| = 0 \quad \text{as } h_{ab} \text{ is Degenerate.}$$

$$\therefore ds^2_\Sigma = \sigma_{AB}\, d\theta^A d\theta^B \;;\; \sigma_{AB} \equiv g_{\alpha\beta}\, e^\alpha_A e^\beta_B$$

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$\sigma_{AB} \equiv$ 2-Tensor $\qquad e^{\alpha}_{A} = \left(\frac{\partial x^{\alpha}}{\partial \theta^{A}}\right)_{\lambda}$

(15) As $h^{\alpha\beta} = h^{ab} e^{\alpha}_{a} e^{\beta}_{b}$

$$h^{\alpha\beta} = g^{\alpha\beta} - U^{\alpha}U^{\beta}$$

$$g^{\alpha\beta} = h^{ab} e^{\alpha}_{a} e^{\beta}_{b} + \epsilon n^{\alpha} n^{\beta}$$

Non Null Case

Why

(16) Null Case

Introduce N^{α} everywhere on Σ, N^{α} (auxillary Null vector)

satisfying $N^{\alpha} k_{\alpha} = +1$

$$N_{\alpha} e^{\alpha}_{A} = 0$$

$$h^{\alpha\beta} = \sigma^{ab} e^{\alpha}_{a} e^{\beta}_{b}$$

$$g^{\alpha\beta} = \sigma^{AB} e^{\alpha}_{A} e^{\beta}_{B} + k^{\alpha} N^{\beta} + N^{\alpha} k^{\beta}$$

(17)
$$\left.\begin{aligned} N^{\alpha} N_{\alpha} &= 0 \\ N^{\alpha} k_{\alpha} &= 1 \\ N^{\alpha} e_{\alpha A} &= 0 \\ N^{\alpha} e_{\alpha B} &= 0 \end{aligned}\right\}$$

4 Condn & 4 variables of N^{α}

$\therefore$ solvable

(18) As by construction we can say $k_{\alpha} e^{\alpha}_{A} = 0$

Let
$$\left.\begin{aligned} k_{\alpha} &= (1,-1,0,0) \\ e_{\alpha 1} &= (0,0,-1,0) \\ e_{\alpha 2} &= (0,0,0,-1) \end{aligned}\right\}$$

H.S. Basis → linear ind. + span

$\therefore k_{\alpha} e^{\alpha}_{A} = 0$

Now let $k_{\alpha} N^{\alpha} = 1 \qquad N_{\alpha} = (1,1,0,0)$

$\therefore$ K_α, N_α

$a(1,-1,0,0)+b(1,1,0,0)=$ They span but they are not linear dual.

(19) As let $k^\alpha = (1,1,\underbrace{0,0})$
for Transverse Direction

Transv. subspace 2D.

$$k^\alpha e_{\alpha A} = 0$$

(20) Integeration on Hyper Surfaces

Non Null Case

$$h_{ab} = \partial_a y^{a'} \partial_b y^{b'} h_{a'b'}$$

$$h = J^2 h'$$

$$d^3y' = J\, d^3y$$

$$d^3y' = \sqrt{\frac{h}{h'}}\, d^3y$$

$$\sqrt{h'}\, d^3y' = \sqrt{h}\, d^3y$$

$\therefore$ $\sqrt{h}\, d^3y = d\Sigma$ is the surface element.

Directed surface element : $n_\mu d\Sigma$

where $n_\mu n^\mu = \epsilon$
Normalized

(21) Null Case

$$h_{ab} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_{ab} & \\ 0 & & \end{pmatrix}$$

$h = 0$ $\therefore$ h_{ab} Degenerate.

n_μ is not Normalized

$n_\mu \longrightarrow \infty$

(22) Therefore till now, we know

in Non Null case $d\Sigma_\mu = n_\mu d\Sigma = n_\mu \sqrt{h}\, d^3y$

in Null Case $\boxed{\text{we donot know yet}}$

(23) $\therefore$ $d\Sigma_\mu = n_\mu d\Sigma$ has to be generalized.

to take Null Case.

Let $d\Sigma_\mu = \epsilon^g_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$

$\epsilon^g_{\mu\alpha\beta\gamma} = \sqrt{-g}\, [\mu\alpha\beta\gamma]$

(24) Non Null case

$d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y \longrightarrow d\Sigma_\mu = n_\mu d\Sigma$

Proof: Claim: $d\Sigma_\mu = f\, n_\mu$

Proof: $d\Sigma_\mu e_1^\mu = d\Sigma_\mu e_2^\mu = d\Sigma_\mu e_3^\mu = 0$

$\therefore$ $d\Sigma_\mu = f\, n_\mu$

(25) Convention: $\epsilon_{\mu\alpha\beta\gamma}\, n^\mu e_1^\alpha e_2^\beta e_3^\gamma > 0$

$= \epsilon_{\mu\alpha\beta\gamma}\, n^\mu \dfrac{\partial x^\alpha}{\partial y^1} \dfrac{\partial x^\beta}{\partial y^2} \dfrac{\partial x^\gamma}{\partial y^3}$

$\underbrace{\epsilon_{\mu\alpha\beta\gamma}}_{\text{sign}} n^{\mu} \frac{\partial x^{\alpha}}{\partial y^1} \frac{\partial x^{\beta}}{\partial y^2} \frac{\partial x^{\gamma}}{\partial y^3}$ depends on the ordering of the 122

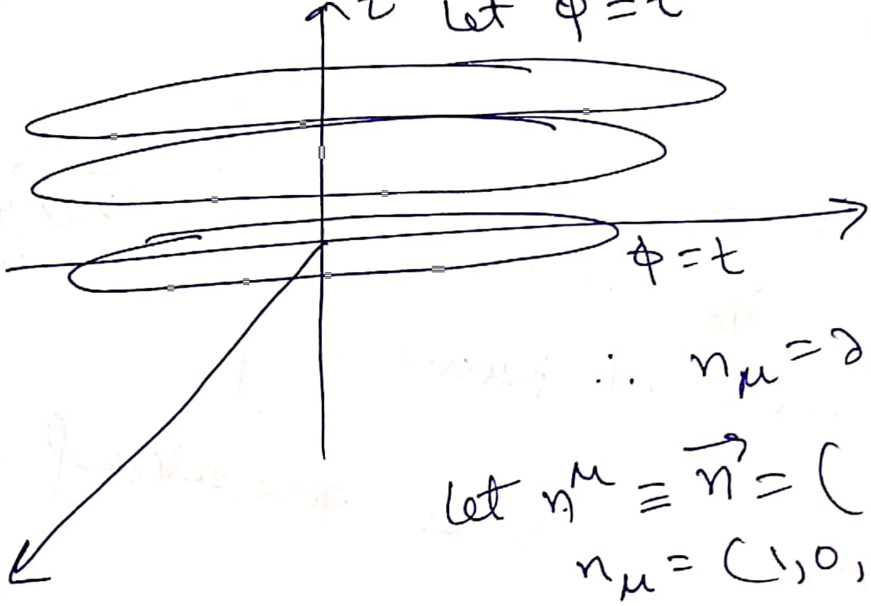
coordinates y^1, y^2, y^3

Convention: Take orderings s.t. $\epsilon_{\mu\alpha\beta\gamma} n^{\mu} e^{\alpha}_1 e^{\beta}_2 e^{\gamma}_3 > 0$

(26) Example: Minkowski Spacetime $\Rightarrow \epsilon_{\mu\alpha\beta\gamma} = [\mu\alpha\beta\gamma]$

$\sqrt{-g} = 1$

Let $\phi = t$



$\therefore n_{\mu} = \partial_{\mu}\phi$ (Convention: $\phi \uparrow$ n_{μ} in that Direction)

Let $n^{\mu} \equiv \vec{n} = (1, 0, 0, 0)$

$n_{\mu} = (1, 0, 0, 0)$

Convention: $\epsilon_{\mu\alpha\beta\gamma} n^{\mu} e^{\alpha}_1 e^{\beta}_2 e^{\gamma}_3 > 0$

$[\mu\alpha\beta\gamma] n^{\mu} e^{\alpha}_1 e^{\beta}_2 e^{\gamma}_3$

Let $y^i = (x, y, z)$

$[\mu 1 2 3] n^{\mu} > 0$

$$d\Sigma_{\mu} = \epsilon_{\mu\alpha\beta\gamma} e^{\alpha}_1 e^{\beta}_2 e^{\gamma}_3 d^3y = f n_{\mu}$$

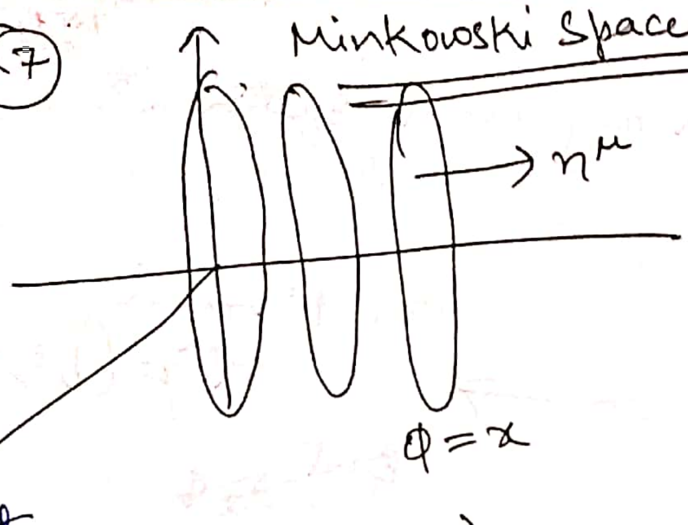
$$f = \epsilon n^{\mu} d\Sigma_{\mu}$$

$$= \epsilon \underbrace{\epsilon_{\mu\alpha\beta\gamma} n^{\mu} e^{\alpha}_1 e^{\beta}_2 e^{\gamma}_3}_{=1} d^3y$$

$$f = \epsilon d^3y = dx\, dy\, dz$$

$$\boxed{d\Sigma_{\mu} = n_{\mu}\, dx\, dy\, dz}$$

(27) Minkowski Spacetime



$n^\mu = \partial^\mu \phi$

But n^μ is purely spatial

$\therefore \; n_\mu = -\partial_\mu \phi$

(convention n^μ in direction of $\uparrow \phi$.)

Let $n_\mu = (0, -1, 0, 0)$

Convention:

$\epsilon_{\mu\alpha\beta\gamma} n^\mu e_1^\alpha e_2^\beta e_3^\gamma > 0$

$[\mu\alpha\beta\gamma] n^\mu e_1^\alpha e_2^\beta e_3^\gamma$

$[\mu 2 3 0] n^\mu = 1 > 0$

let $y^i = (y, z, t)$

$\Rightarrow \frac{\partial x^\alpha}{\partial y^1} = \frac{\partial x^\alpha}{\partial y} = (0,0,1,0)$

$d\Sigma_\mu = f n_\mu$

$f = \epsilon d^3y = -dy\,dz\,dt$

$\boxed{d\Sigma_\mu = -n_\mu\, dy\, dz\, dt}$ $\rightarrow$ But $d\Sigma_\mu = n_\mu d^3y$ so Be Cautious here

$\rightarrow$ for Timelike H.S. $d\Sigma_\mu$ in opposite direction of n_μ

Proof: Non Null Case $d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma} e_1^\alpha e_2^\beta e_3^\gamma d^3y$

$\downarrow$

$d\Sigma_\mu = n_\mu \sqrt{h}\, d^3y$

(28) As we Know

$d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma} e_1^\alpha e_2^\beta e_3^\gamma d^3y$

$e_i^\mu d\Sigma_\mu = 0 \quad \forall \; i = 1, 2, 3.$

$\therefore \; d\Sigma_\mu = f n_\mu \quad \text{—} (1)$

And Also $\epsilon_{\mu\alpha\beta\gamma} e_1^\alpha e_2^\beta e_3^\gamma = g n_\mu$

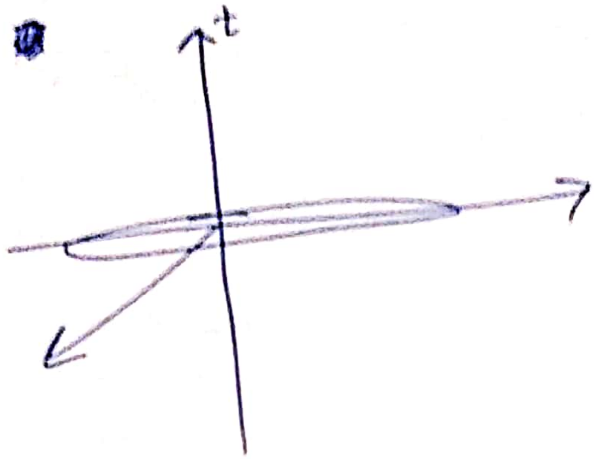
$g = \epsilon\, \epsilon_{\mu\alpha\beta\gamma} n^\mu e_1^\alpha e_2^\beta e_3^\gamma$

As g is scalar

$\therefore$ we can evaluate in any coord. system x^α.

let $\phi = x^0$

$y^i = x^i$



Normal

$n^\mu \neq (1,0,0,0)$

~~But $\partial_\alpha \phi$~~

$$\therefore g \stackrel{*}{=} \epsilon\, \epsilon_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_1 e^\beta_2 e^\gamma_3$$

let $y^i = (x, y, z)$ for $\epsilon_{\mu\alpha\beta\gamma} n^\mu e^\alpha_1 e^\beta_2 e^\gamma_3 > 0$

$$g \stackrel{*}{=} (+1)\sqrt{-g}\,[\mu\, 1\, 2\, 3]\, n^\mu$$

$$g \stackrel{*}{=} \sqrt{-g}\, n^0$$

$$g^{00} \stackrel{*}{=} \partial_\alpha \phi\, \partial^\alpha \phi \stackrel{*}{=} g^{\alpha\beta} \partial_\alpha \phi\, \partial_\beta \phi \quad ??$$

$$n_\mu = \frac{\epsilon\, \partial_\mu \phi}{|\partial_\alpha \phi\, \partial^\alpha \phi|^{1/2}}$$

$$n_0 = \epsilon\, |\partial_\alpha \phi\, \partial^\alpha \phi|^{-1/2} = \epsilon\, |g^{00}|^{-1/2}$$

$$n^0 = g^{0\alpha} n_\alpha \stackrel{*}{=} g^{00} n_0 \stackrel{*}{=} g^{00} \frac{\epsilon}{|g^{00}|^{-1/2}}$$

$g^{00} > 0$

$\Uparrow$

$\partial_\alpha \phi\, \partial^\alpha \phi > 0 \quad \Leftarrow$

As n_α is Time like

$\partial_t \phi > 0$

$\therefore \partial^\alpha \phi = (+, 0, 0, 0)$

$\partial_\alpha \phi = (+, 0, 0, 0)$

$\therefore n^0 \overset{*}{=} (g^{00})^{1/2}$

$g \overset{*}{=} \sqrt{-g}\,(g^{00})^{1/2}$

$g^{00} = \dfrac{\text{cif } g_{00}}{|g|} = \dfrac{|h|}{|g|}$?? $\rightarrow$ as $h_{ab} = g_{\alpha\beta} e^{\alpha}_{a} e^{\beta}_{b}$

$g_{\alpha\beta} = \begin{pmatrix} \; & \; \\ \; & h_{ab} \end{pmatrix}$

as $A^{-1} = \dfrac{\text{adj } A}{|A|}$

$\therefore g \overset{*}{=} \sqrt{-g}\,\dfrac{|h|^{1/2}}{|g|^{1/2}} \overset{*}{=} |h|^{1/2}$

$\Rightarrow$ from ①

$\epsilon_{\mu\alpha\beta\gamma}\, e^{\alpha}_{1} e^{\beta}_{2} e^{\gamma}_{3} = h^{1/2} n_{\mu}$

$\therefore d\Sigma_{\mu} = |h|^{1/2} n_{\mu}\, d^3y$

(29) <u>Null Case</u>

Choosing our usual coordinates on null surface $y^i = (\lambda, \theta^A)$

$e^{\alpha}_{1} = k^{\alpha} \qquad e^{\alpha}_{2,3} = \left(\dfrac{\partial x^{\alpha}}{\partial \theta_{A,B}}\right)$

$d^3y = d\lambda\, d^2\theta$

$d\Sigma_{\mu} = \epsilon_{\mu\alpha\beta\gamma}\, k^{\alpha} e^{\beta}_{2} e^{\gamma}_{3}\, d\lambda\, d^2\theta$ (Valid for any coordinate system.)

$= k^{\alpha}\, dS_{\mu\alpha}\, d\lambda$

$dS_{\mu\alpha} = \epsilon_{\mu\alpha\beta\gamma}\, e^{\beta}_{2} e^{\gamma}_{3}\, d^2\theta$

$dS_{\mu\alpha} \equiv$ 2 Dim Surface Area

Imp $d\Sigma_{\mu} = \epsilon_{\mu\alpha\beta\gamma}\, e^{\alpha}_{1} e^{\beta}_{2} e^{\gamma}_{3}\, d^3y$

Valid for any coordinate system & Any T/S/N case

(30) Properties of $dS_{\mu\alpha}$

(1) $dS_{\mu\alpha}$ is antisym. in μ & α.

(2) $dS_{\mu\alpha}$ is orth. to e^{α}_{A}, e^{α}_{B}

$\therefore$ $dS_{\mu\alpha}$ in the k & N

But as $dS_{\mu\alpha}$ is ant. in μ α

$\therefore$ $dS_{\mu\alpha} \propto \cancel{\beta}\,[k_{\mu} N_{\alpha} - k_{\alpha} N_{\mu}]$

let $\epsilon_{\mu\alpha\beta\gamma} e^{\beta}_{2} e^{\gamma}_{3} = 2f\, k_{[\mu} N_{\alpha]}$

(31) Moreover,

2D submanifold in 4D needs 2 Normals to be defined

See L-(10)

$\boxed{26, 27}$

(32) What about
- 1D submanifold in 3D?
- 2D submanifold in 3D?

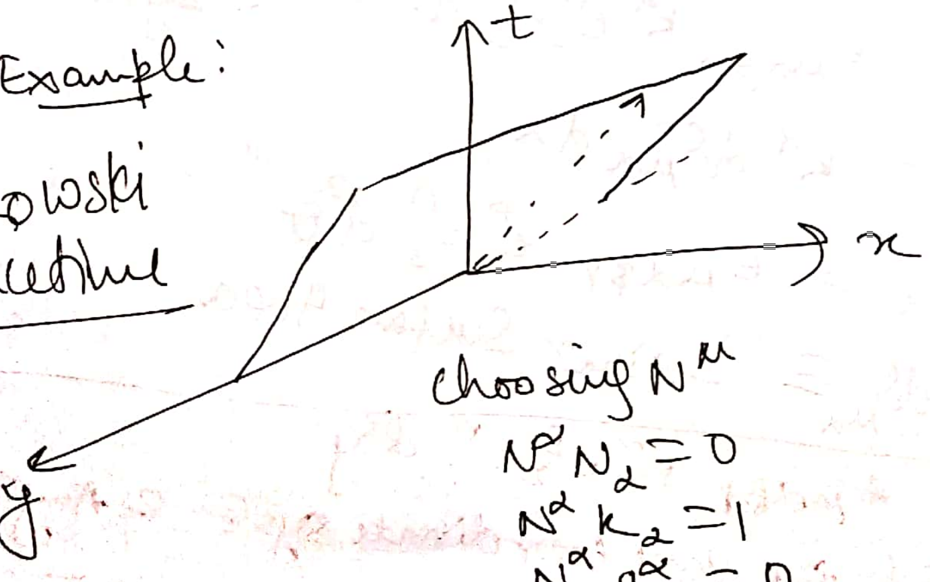
(33) <u>Convention</u>: $\epsilon_{\mu\alpha\beta\gamma} N^{\mu} k^{\alpha} e^{\beta}_{2} e^{\gamma}_{3} > 0$

i.e. the ordering of θ^{A} should be s.t.

$$\epsilon_{\mu\alpha\beta\gamma} N^{\mu} k^{\alpha} \frac{\partial x^{\beta}}{\partial y^{2}} \frac{\partial x^{\gamma}}{\partial y^{3}} > 0$$

(34) <u>Example</u>:

<u>Minkowski Spacetime</u>



$\phi = t - x = \text{const}$

$k_{\alpha} = \partial_{\alpha}\phi$

$k_{\alpha} = (1, -1, 0, 0)$

$k^{\alpha} = (1, 1, 0, 0)$

Choosing N^{μ}

$N^{\alpha} N_{\alpha} = 0$

$N^{\alpha} k_{\alpha} = 1$

$N^{\alpha} e^{\alpha}_{A} = 0$

let $N_\alpha = \frac{\partial_\alpha (t+x)}{2}$

$= \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$

$N^\alpha = \left(\frac{1}{2}, \frac{-1}{2}, 0, 0\right)$

$\epsilon_{\mu\alpha\beta\gamma} N^\mu \kappa^\alpha e_2^\beta e_3^\gamma > 0$

$= [\mu\alpha\beta\gamma] N^\mu \kappa^\alpha e_2^\beta e_3^\gamma$

$= \frac{[0\alpha\beta\gamma]}{2} \kappa^\alpha e_2^\beta e_3^\gamma - \frac{[1\alpha\beta\gamma]}{2} \kappa^\alpha e_2^\beta e_3^\gamma$

$= \frac{[01\beta\gamma]}{2} e_2^\beta e_3^\gamma - \frac{[10\beta\gamma]}{2} e_2^\beta e_3^\gamma$

If $\theta^A = (y, z)$

$= \frac{[0123]}{2} - \frac{[1023]}{2}$

$= \frac{1}{2} + \frac{1}{2} = 1 > 0.$

$\sigma'_{AB} = \partial_{A'} \theta_A \partial_{B'} \theta_B \sigma_{AB}$

$\sigma' = J^2 \sigma$

$J = \sqrt{\frac{\sigma'}{\sigma}}$

$\Rightarrow dS_{tx} = dy\, dz = -dS_{xt}$

$d\Sigma_t = dt\, dy\, dz = -d\Sigma_x$

$\therefore \sqrt{\sigma}\, d^2\theta$

(35) As $dS_{\mu\alpha} = \epsilon_{\mu\alpha\beta\gamma} e_2^\beta e_3^\gamma d^2\theta$

we will show $\boxed{dS_{\mu\alpha} = 2 \kappa_{[\mu} N_{\alpha]} \sqrt{\sigma}\, d^2\theta}$

N_α is the Auxillary Null vector → gives direction

But we know $d\Sigma_\mu = \kappa^\beta dS_{\mu\beta} d\lambda$

$\Rightarrow d\Sigma_\mu = 2\kappa^\beta \kappa_{[\mu} N_{\beta]} \sqrt{\sigma}\, d^2\theta\, d\lambda = \kappa_\mu \sqrt{\sigma}\, d^2\theta\, d\lambda$

$\therefore$ for Null case

$$d\Sigma_\mu = k_\mu \sqrt{\sigma}\, d^2\theta\, d\lambda$$

for Non Null case

$$d\Sigma_\mu = n_\mu \sqrt{|h|}\, d^3y$$

$\sqrt{\sigma}\, d^2\theta$: Element of 2 Dim. cross sectional area

$\sqrt{|h|}\, d^3y$: Element of 3 Dim cross sectional area

(36)

(37) ~~As we k~~

Proof: $dS_{\mu\alpha} = 2\, k_{[\mu} N_{\alpha]} \sqrt{\sigma}\, d^2\theta$

As

$\epsilon_{\mu\alpha\beta\gamma}\, e_2^\beta e_3^\gamma = 2f\; k_{[\mu} N_{\alpha]}$

where $f = \epsilon_{\mu\alpha\beta\gamma} N^\mu k^\alpha e_2^\beta e_3^\gamma > 0$

Let us choose a coordinate system to evaluate f as f is scalar $\therefore$ we can evaluate it in any system.

Let $x^a = y^a = (\lambda, \theta^A)$ on Σ

$x^0 = t$

$k^\alpha = \dfrac{dx^\alpha}{d\lambda}$ $\quad k^\alpha N_\alpha = 1$ (for choosing 2D submanifold)

$k^\alpha = (1, 1, 0, 0)$ (How?)

$N_0 + N_1 = 1$

$N_0^2 - N_1^2 - N_2^2 - N_3^2 = 0$ (as $N^\alpha N_\alpha = 0$) $\Big\}$ (*)

But $N_\alpha e_A^\alpha = 0$

$\cancel{N_2 \dfrac{\partial x^2}{\partial\theta_1} + N_3 \dfrac{\partial x^3}{\partial\theta_2} = N_2 + N_3}$

$N_0 \dfrac{\partial x^0}{\partial\theta_A} + N_1 \dfrac{\partial x^1}{\partial\theta_A} + N_2 \dfrac{\partial x^2}{\partial\theta_A} + N_3 \dfrac{\partial x^3}{\partial\theta_A} = N_2 = 0$

Similarly $N_3 = 0$

$\therefore$ $N_0 + N_1 = 1$

$N_0^2 - N_1^2 = 0$

$N_0 - N_1 = 0$

$N_0 = 1/2 \quad N_1 = 1/2$

$\Rightarrow N^\alpha = (1/2, -1/2, 0, 0)$

$$37 \quad 0 = k^{\alpha} k_{\alpha} = g_{\alpha\beta} k^{\alpha} k^{\beta} = g_{\alpha\beta} \partial^{\alpha}\phi \, \partial^{\beta}\phi$$
$$= g^{\alpha\beta} \partial_{\alpha}\phi \, \partial_{\beta}\phi$$

(38) Gauss-Stokes Theorem

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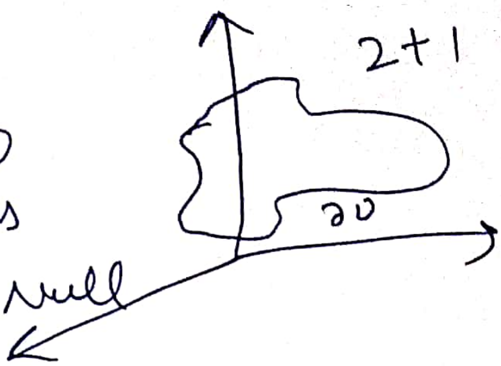

in Region of V in manifold
Bounded by closed H.S. ∂V
The H.S. may have segments
that are timelike, spacelike or Null

$$\int_V \nabla_\alpha A^\alpha \sqrt{-g}\, d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$$

∇A^α where $d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma d^3y$