① Cosmology

FLRW metric

① Metric of spatial homogeneous & spatial isotropic space

cylinder - homo but not isotropic

isot but not homo

Both imply at any pt. there is isotropy

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

All symmetries apply on this

$$= g_{00}\, dt^2 + 2 g_{0i}\, dx^i dt - h_{ij}\, dx^i dx^j \quad ①$$

Comoving coordinate Distance

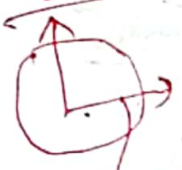
eg. sphere — Maximally symmetric spatial Geometry

(i) Isotropy

Transformation of spatial coordinates as $x \rightarrow -x$

$$ds^2 = g_{00}\, dt^2 - 2 g_{0i}\, dx^i dt - h_{ij}\, dx^i dx^j \quad ②$$

Comoving coordinate



coordinate also expands s.t. coordinates B/w 2 objects Remain at same Distance

Adding ① & ②

$$ds^2 = g_{00}\, dt^2 - h_{ij}\, dx^i dx^j$$

$$\therefore g_{0i} = 0$$

(ii) $d\tau = \sqrt{g_{00}}\, dt$ (using this transform)

$$ds^2 = d\tau^2 - h_{ij}\, dx^i dx^j$$

Writing t instead of τ

$$ds^2 = dt^2 - h_{ij}\, dx^i dx^j$$

Doubt: isotropy → spherical symmetry ?

iii) Isotropy

$$d\Sigma^2 = h_{ij}\, dx^i dx^j$$

Due to isotropy

Spherical symmetry i.e. there one pt. in one direction isn't different from other i.e. $dx^i dx^j = dx + dy \ldots$

&

due to ~~sym~~ isotropy

$$\boxed{dx\,dy = 0}$$

& due to isotropy $h_{ij}(\ldots) = h(r)\,a(t)$

$$\therefore \; d\Sigma^2 = a^2(t)\,\lambda^2(r)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Taking $\lambda r = r'$ ~~or~~ & redefing $\lambda' = \dfrac{\lambda}{r\frac{d\lambda}{dr} + \lambda}$ (to make λ appear only once)

we get

$$d\Sigma^2 = a^2\left[\lambda'^2 dr'^2 + r'^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

just as $dt = f(\tau)\,d\tau$ was made

This was all due to isotropy

Now imposing Homogeneity → why not 3D?

(iv) ∴ All pts should be same.

We now ~~seek~~ metric that describes HS immersed in a spherical (4-D) Eculidean space. → No time evolution

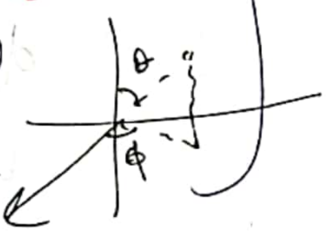
fixed sphere

$$a^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 \quad \text{— (2)}$$

$$\left\{\begin{aligned} x_1 &= a\cos\chi\,\sin\theta\,\sin\phi \\ x_2 &= a\cos\chi\,\cos\theta \\ x_3 &= a\cos\chi\,\sin\theta\,\cos\phi \\ x_4 &= a\sin\chi \end{aligned}\right.$$

in Company

$$\left\{\begin{aligned} x &= a\sin\theta\sin\phi \\ y &= a\sin\theta\cos\phi \\ z &= a\cos\theta \end{aligned}\right.$$



Diff (3)

$$x_4\,dx_4 = -(x_1 dx_1 + x_2 dx_2 + x_3 dx_3)$$

as ~~$ds^2 = dx_1^2 + dx_2^2 + \ldots$~~

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$$

$$= dx_1^2 + dx_2^2 + dx_3^2 + \frac{(x_1 dx_1 + x_2 dx_2 + x_3 dx_3)^2}{x_4^2}$$

$dx_1 = -a\sin\chi\sin\theta\sin\phi\, d\chi + a\cos\chi\cos\theta\sin\phi\, d\theta$
$+ a\cos\chi\sin\theta\cos\phi\, d\phi$

$dx_2 = -a\sin\chi\cos\theta\, d\chi - a\cos\chi\sin\theta\, d\theta$

$dx_3 = -a\sin\chi\sin\theta\cos\phi\, d\chi + a\cos\chi\cos\theta\cos\phi\, d\theta$
$-a\cos\chi\sin\theta\sin\phi\, d\phi$

$dx_4 = a\cos\chi\, d\chi$

$ds^2 = a^2\left(d\chi^2 + \sin^2\chi\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right)$

$\rightarrow d\Sigma^2 = d^2\left(\lambda'^2 dr'^2 + r'^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right)$

Both should be same

$\therefore \left.\begin{array}{l}\sin\chi = r' \\ d\chi = \lambda' dr'\end{array}\right\}$ $\cos\chi\, d\chi = dr'$

$\lambda' = \dfrac{1}{\cos\chi}$

$\boxed{\lambda' = \dfrac{1}{\sqrt{1-r'^2}}}$

$\boxed{\begin{array}{l}\lambda r = r' \\ \lambda' = \dfrac{\lambda}{r\frac{d\lambda}{dr} + \lambda}\end{array}}$

why k should be discrete?

(v) Generalizing (iv)

$a^2 = x_1^2 + x_2^2 + x_3^2 + k x_4^2$

Going with same story we obtain

$ds^2 = a^2\left(d\chi^2 + F^2(\chi)\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right)$

where $F(\chi) = \begin{cases}\sin\chi & k = 1 \\ \chi & k = 0 \\ \sinh\chi & k = -1\end{cases}$

This metric is again $\equiv d_2^2$

$ds^2 = d_2^2$

$\therefore$ By equating $\lambda' = \dfrac{1}{\sqrt{1-kr'^2}}$

Conclusion

Finally, we obtain

(1)
$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

FLRW metric

we have use r, λ here in practice $\lambda = \lambda'$ & $r = r'$ here

(2) Cartesian form

$$ds^2 = c^2 dt^2 - a^2\lambda^2\left(dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right)$$

$\lambda r = r'$

$$\lambda' = \frac{\lambda}{r\frac{d\lambda}{dr} + \lambda}$$

$$\lambda' = \frac{1}{\sqrt{1-kr'^2}}$$

$$\frac{\lambda}{r\frac{d\lambda}{dr} + \lambda} = \frac{1}{\sqrt{1-k\lambda^2 r^2}}$$

$$\lambda(1-k\lambda^2 r^2) = r^2\left(\frac{d\lambda}{dr}\right)^2 + \lambda^2 + 2\lambda r\frac{d\lambda}{dr}$$

$$\lambda - k\lambda^3 r^2 = r^2\left(\frac{d\lambda}{dr}\right)^2 + 2\lambda r\frac{d\lambda}{dr} + \lambda^2$$

$$\Longrightarrow \quad \lambda = \frac{1}{1+\frac{kr^2}{4}}$$

$$\therefore\ ds^2 = dt^2 - \frac{a^2}{\left(1+\frac{kr^2}{4}\right)^2}\left[dx^2+dy^2+dz^2\right].$$ (5)

(3) $ds = 0$ (Null geodesic)

$$c^2dt^2 = a^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)\right]$$

Assuming $d\theta = d\phi = 0$

$$c\,dt = \frac{a\,dr'}{\sqrt{1-kr'^2}} = \frac{a\,dr'}{1-\frac{kr'^2}{2}}$$ → for small propagation distances

r' small

This is not Real Distance

$\therefore\ 1-\frac{kr^2}{2} \approx 1$

$z << 1$

Physical Distance = $a(t)$ Coord. Dist.

$$\therefore\ \boxed{c\,dt \approx a\,dr}$$

(4) $c\,dt = D \approx a\,dr'$ → Hubble law

D = Distance travelled by light (Null geodesic) (small propagation)

Not as coordinate distance is fixed

$\dot{D} = \dot{a}\,dr'$ (as r, θ, ϕ, t are independent $\therefore \frac{dr}{dt} = 0$)

(2) $\dot{D} = HD$

$H = \frac{\dot{a}}{a}$ = Hubble Constant
= rate of expn of Space

Hubble law apply to any system that expands in homog & isotropic way.

as $\lambda r = r'$

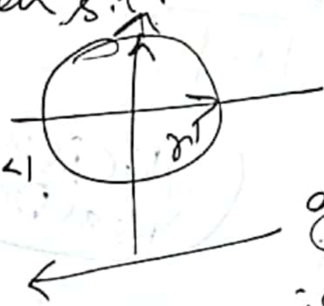
r

Hubble law applies only to small Distances in any K-space or to only flat space at any distance

(r', θ, ϕ, t)

(5) Physical Distance Vs Comoving Distance

Coordinate system is choosen s.t. it is expanding at same rate as Universe.



$\therefore$ Comoving Distance b/w any two galaxies moving away from each other one is constant.

Assuming for small Distance or $z \ll 1$.

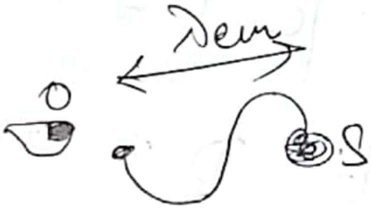
$\dot{D} = \dot{a}\,dr' = HD$

$D = a(t)\,dr'$

D → Physical Distance

dr' → Comoving distance

(6) Red Shift



S $\lambda_{em} = c\,dt$

dt → time between crests

Assuming $v \ll c$

$$\frac{d\lambda}{\lambda} = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{v}{c} = z$$

Source moves away in dt time at v speed i.e. vdt $\therefore \lambda_0 = c\,dt + vdt$

(3) $$\boxed{z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}}$$

If the source is following Hubble law $\dot{D} = HD \equiv v = HD \Rightarrow \frac{d\lambda}{\lambda} = \frac{v}{c} = +H\,dt = +\frac{da}{a}$

$dt = t_{obs} - t_{em} = +ve$

$\frac{d\lambda}{\lambda} = +\frac{da}{a} \Rightarrow \ln\lambda\Big|_{em}^{obs} = +\ln a\Big|_{em}^{0} \Rightarrow \ln\frac{\lambda_{obs}}{\lambda_{em}} = +\ln\frac{a_0}{a_{em}}$

Energy Momentum Conservation (1) Continuity Equation

(P)

(7) Conservation of Mass $\Longrightarrow$ $\partial_i J^i = 0$

$\Downarrow$

Continuity Eqn

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{v}) = 0 \qquad J^i = (\rho, \rho \vec{v})$$

$$\Downarrow$$

$$\boxed{\frac{d\rho}{dt} = -\rho\, \vec{\nabla}\cdot\vec{v}}$$

$\nabla_\alpha T^{\alpha\beta} = 0$ (Energy Mom. Conservation)

$$\nabla_\alpha \left((p+\rho) U^\alpha U^\beta - p g^{\alpha\beta} \right) = 0$$

$$U^\beta (p+\rho) \nabla_\alpha U^\alpha + (p+\rho) U^\alpha \cancel{\nabla_\alpha U^\beta} - \cancel{p \nabla_\alpha g^{\alpha\beta}} - g^{\alpha\beta} \nabla_\alpha p = 0$$

$$(p+\rho) U^\beta \nabla_\alpha U^\alpha = g^{\alpha\beta} \nabla_\alpha p$$

$$\boxed{(p+\rho) \nabla_\alpha U^\alpha = U^\alpha \nabla_\alpha p = U^\alpha \partial_\alpha p}$$

This is very important reln as it relates easily calculatable z with main cosmol. fn $a(t)$

At long distances $v \ll c$ is not valid

Also $v \ll c$ is assumed

Only valid small dist

let $a_0 = 1$

$$\boxed{\frac{\lambda_{obs}}{\lambda_{em}} = (a_{em})^{-1}} \Rightarrow (a_{em}^{-1} - 1) = z \Rightarrow \boxed{a_{em}^{-1} = 1+z}$$

(8) $T^{00} = \rho$ Restframe $u^i = (t, 0, 0, 0)$

$T^i_i = \rho - 3p$

$T^{ij} = (p+\rho)u^i u^j - p g^{ij}$ $\quad i = 1, 2, 3$

$= (p+\rho)0 - \frac{p h^{ij}}{a^2} = -\frac{p h^{ij}}{a^2}$

$g^{ij} = \frac{h^{ij}}{a^2}$

(9) $\nabla_\alpha T^{\alpha\beta} = \partial_\alpha T^{\alpha\beta} + \Gamma^\alpha_{\alpha\gamma} T^{\gamma\beta} + \Gamma^\beta_{\alpha\gamma} T^{\alpha\gamma}$

$\Gamma^i_{0j} = \Gamma^i_{j0} = \frac{\dot a}{a}\delta^i_j$

$\Gamma^0_{ij} = a\dot a h_{ij}$ $\quad g^{ij} = \frac{h^{ij}}{a^2}$

$\cancel{\Gamma^i_{jk} = }$

$\nabla_\alpha T^{\alpha 0} = \partial_\alpha T^{\alpha 0} + \Gamma^0_{\alpha\gamma} T^{\alpha\gamma} + \Gamma^\alpha_{\alpha\gamma} T^{\gamma 0}$

$= \frac{\partial \rho}{\partial t} + \Gamma^0_{ij} T^{ij} + \Gamma^\alpha_{\alpha 0} T^{00}$

$= \frac{\partial \rho}{\partial t} + a\dot a h_{ij}(-p g^{ij}) + \frac{\dot a}{a} 3 \rho$

For Radiation $= \frac{\partial \rho}{\partial t} + \frac{3\dot a p}{a} + \rho \frac{3\dot a}{a}$

$p = \frac{\rho}{3}$ $\quad 0 = \frac{\partial \rho}{\partial t} + \frac{4\dot a}{a}\rho \Rightarrow \frac{d}{dt}(\rho a^3) = 0$

$$\rho a^4 = \epsilon = \text{const}$$

$$\rho = \frac{\epsilon}{a^4}$$

$$\frac{\partial \rho}{\partial t} + 3\frac{\dot{a}}{a}(p+\rho) = 0$$

$$\frac{\partial \rho}{\partial t} + 3H(p+\rho) = 0$$

(10) Define Spatial Curvature Scalar P

$$P = h^{ij} P_{ij}$$

(11) Taking spatial comp. of E-M Tensor

$\Downarrow$

Constant Curvature.

Taking Λ also

$$\frac{\partial\left(\rho + \frac{\Lambda}{8\pi G}\right)}{\partial t} + a\dot{a}h_{ij}\left(\frac{\Lambda g^{ij}}{8\pi G} - p g^{ij}\right) + \frac{3\dot{a}}{a}\left(\rho + \frac{\Lambda}{8\pi G}\right)$$

$$\frac{\partial \rho}{\partial t} + a\frac{\dot{a}}{a}\left(\frac{-3\Lambda}{8\pi G} + 3p\right) + \frac{3\dot{a}}{a}\left(\rho + \frac{\Lambda}{8\pi G}\right) = 0$$

(8) $$\boxed{\frac{\partial \rho}{\partial t} + 3H(p+\rho) = 0}$$

Cty Eqn Invariant

Expanding Universe

① $R_{\alpha\beta} - \frac{g_{\alpha\beta}}{2} R = T_{\alpha\beta}\, 8\pi$

$$\boxed{R_{00} - \frac{g_{00}}{2} R = T_{00}\, 8\pi G} \quad \text{--- ①}$$

$$R^{\alpha}_{\alpha} - gR = -R = 8\pi T$$

$$\boxed{R = -8\pi T} \quad \text{--- ②}$$

Taking the FRW metric

$$ds^2 = dt^2 - a^2\left[\frac{dr^2}{1-kr^2} + r^2\{d\theta^2 + \sin^2\theta\, d\phi^2\}\right]$$

Calculating Γ, R & using ① & ② we obtain

Using $R_{\alpha\beta} + \frac{g_{\alpha\beta}}{2} R = 8\pi T_{\alpha\beta}$

⑥
$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Freidman Eq^n

②) In the previous section we have also obtained conservation Eq^n $\nabla_\alpha T^{\alpha 0} = 0$

$$\frac{\partial \rho}{\partial t} + 3H(p+\rho) = 0$$

Freidman Eq^n & conserv. Eq^n are not ind.,
Diff ①st Freidm & using conser Eq, we obtain ②nd Freid

(11)

③ Critical Density $\rho_c = \frac{3H_0^2}{8\pi G}$

Density parameter $\Omega = \frac{\rho}{\rho_c}$

→ Units of Critical density

Given an expression for critical density assuming $H_0 = 71$ km/sec/Mpc critical density can be calculated in different units like Jm^{-3}, $GeV m^{-3}$ etc.

$\boxed{1J = 6.24 \times 10^{18} eV}$

using 1st Freidman Eqn-

$$1 = \frac{8\pi G\rho}{3H^2} - \frac{k}{a^2H^2} = \frac{\rho}{\rho_c} - \frac{k}{a^2H^2}$$

let $G = 1$

$$1 = \Omega - \frac{k}{a^2H^2}$$

at any time ← Density component

$$\boxed{1 = \Omega(a) + \Omega_k(a)} \quad (7)$$

Curvature component. $\Omega_k(a) \equiv \frac{-k}{a^2H^2}$

④ for $k=0$ flat FLRW

$\Omega = 1 \Rightarrow \rho = \rho_c$ ~~Doubt~~

for $k=+1$ $\Omega > 1$ for $k=-1$ $\Omega < 1$

$$\Omega_k = \frac{-k}{a^2H^2} = \frac{\rho_k}{\rho_{k,c}}$$

$$\frac{-k3}{a^2 8\pi G} = \rho_k$$

$$\rho_k = \frac{-3k}{8\pi G}$$

⑤ Non Relativistic fluid

$p = 0$ [Dust matter (galaxy) Non Interacting particles with nonrelativistic vel. (CDM)]

Also Pressure of free particles with mean sq. vel. v $p = nmv^2 << \rho = nmc^2$ for $v << c$. By Cont. Eq. $\frac{\dot\rho}{\rho} = -3\frac{\dot a}{a}$

$1 = \Omega(a) + \Omega_k(a)$ at every Epoch

This Relation extends to models with diff comp.

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(5)

$$\ln \rho = -3 \ln a$$

$$\rho \propto a^{-3} \Rightarrow \frac{\rho}{\rho_0} = \left(\frac{a_0}{a}\right)^3$$

$$\boxed{\rho = \rho_0 \left(\frac{a_0}{a}\right)^3}$$

Let $a_0 = 1$

$$\rho = \frac{\rho_0}{a^3} = \rho_0 (1+z)^3 = \rho_c \Omega_{NR} (1+z)^3$$

$\boxed{\text{Present value}}$ $= \Omega_{NR} = \frac{\rho_0}{\rho_c}$

$k=0$; $\Omega_m = 0$; $\Omega_\Lambda = 0$; $a(t) = \left(\frac{t}{t_0}\right)^{1/2}$
Calculate
(1) Age of Universe
(2) Comoving distance of object at z
(3) Com. Horizon distance

(6) By F 1st Eq.

$k=0 \quad H_0^2 = \frac{8\pi G}{3} \rho_0 \rightarrow$ Flat space

also $H^2 = \frac{8\pi G}{3} \rho$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{H_0^2}{\rho_0} \rho = H_0^2 a_0^3 a^{-3} = H_0^2 a^{-3}$$

Why can't we do same for Radiation

$$\dot a^2 = \frac{H_0^2}{a} \Rightarrow a^{1/2} da = H_0 dt$$

$$\frac{1}{2} a^{3/2} = H_0 t$$

Assuming flat space

$$\boxed{a = (2H_0)^{2/3} t^{2/3}}$$

H_0 value we know

$\therefore$ By $H_0^2 = \frac{8\pi G}{3} \rho_0$ we can find present matter Density $\Rightarrow$

(7) $H_0 = 100h \frac{km}{s \cdot Mpc}$ $\qquad h = 0.70 \pm 0.04$

(8) Relativistic Component

$p = \frac{\rho}{3}$ (A photon gas distributed as Black body has pressure $p = \frac{\rho}{3}$)

cty Eq $\quad \dot{\rho} + H 4 \rho = 0$

$\frac{\dot{\rho}}{\rho} = -4 \frac{\dot{a}}{a} \Rightarrow \ln \rho = -4 \ln a$

$\rho \propto a^{-4}$

By cty Eqn $\rho = f(a) = f(x)$

$$\boxed{\rho = \rho_0 \left(\frac{a_0}{a}\right)^4}$$

let $a_0 = 1 \Rightarrow \rho_R = \frac{\rho_0}{a^4} = \rho_c \Omega_R (1+z)^4$ (current value)

$\rho_m \propto a^{-3}$, $\rho_\gamma \propto a^{-4}$ } at Early time Radiation Dominant

$\rho_{NR} = \rho_c \Omega_{NR} (1+z)^3$

Radiation Density dilutes as a^{-3} bec. of vol. expn & a^{-1} bec. of Energy Redshift.

(9) Photongas in Eqm with matter has energy Density $\rho_\gamma = \frac{g\pi^2}{30} T^4$ $\left\{ \begin{array}{l} g = 2 \text{ for photons} \\ g = 3.36 \text{ 3 massless Neutrino.} \end{array} \right\}$

$\rho_\gamma \propto a^{-4} \quad \therefore T \propto \frac{1}{a} \Rightarrow$ as $T_0 = 3K$

$\rho_\gamma = g\, 2.3\, 10^{-34} \text{ g cm}^{-3}$

$\rho_{0,m} = 2 \times 10^{-29} h^2 \text{ g cm}^{-3}$

$\rho_{0,m} > \rho_{\gamma,0}$ [Matter Dominated Epoch]

(10) $\rho_m = \rho_{m,0}\, a^{-3}$
$\rho_r = \rho_{r,0}\, a^{-4}$

Equivalence Epoch a_e where $\rho_r = \rho_m$

$$a_e = \frac{\rho_{r,0}}{\rho_{m,0}} = \frac{\rho_{m,c}\,\rho_{r,0}}{\rho_{r,c}\,\rho_{m,0}} = \frac{\Omega_r}{\Omega_m}$$

assuming $\rho_{r,c} = \rho_{m,c}$

$\boxed{\text{Equivalence Redshift}}$?

at late time $a >> 1$
const $= \rho_\Lambda$ is Dominating

Vacuum $w = -1$
$\rho \propto$ const $\quad p = -\rho$
$\rho =$ const

By Cty Eq^n

(11) $p = w\rho$

$$\rho \propto a^{-3(w+1)}$$

$$\boxed{\begin{array}{ll} p = 0 & \rho \propto a^{-3} \\ p = \frac{\rho}{3} & \rho \propto a^{-4} \\ p = w\rho & \rho \propto a^{-3(w+1)} \end{array}}$$

(12) Assuming $k = 0$
By (1)st F Eq^n $\Rightarrow$ $H^2 = \frac{8\pi G}{3}\rho$

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = H_0^2\, a^{-3(w+1)}$$

$$\dot a^2 = H_0^2\, a^{-3w-1}$$

$$a^{\frac{3w+1}{2}}\, \dot a = H_0 \Rightarrow \left(\frac{3w+1}{2}\right) a^{\frac{3w-1}{2}} = H_0 t$$

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(13) $H^2 = \frac{8\pi G}{3}\left(\rho_{m,0}\, a^{-3} + \rho_{\gamma,0}\, a^{-4} + \rho_{K,0}\, a^{-2}\right)$ (18)

$\rho_c = \frac{3H_0^2}{8\pi G}$ $\Omega = \frac{\rho}{\rho_c}$ $\rho_{K,0} = -\frac{3k}{8\pi G}$

(8) $$\boxed{H^2 = H_0^2\left(\Omega_{m,0}\, a^{-3} + \Omega_{\gamma,0}\, a^{-4} + \Omega_{K,0}\, a^{-2}\right)}$$

s.t. $\Omega_{m,0} + \Omega_{\gamma,0} + \Omega_{K,0} = 1$

Every other component can be added when its behaviour with a is known.

(14) Qualitative Trends

→ $\rho + 3p > 0$ till Now

By 2^{nd} F Eqn $\ddot{a} < 0$ Decelerating.

∴ (1) $a = 0$ at some time t_{sing}. (if initially $\dot{a} > 0$)

Let $\dot{a} = \text{Const} = k \Rightarrow a(t) = a_0 + k(t - t_0) = a_0 + \dot{a}(t - t_0)$

→ Proves Big Bang

$\Rightarrow a \propto t^{\frac{2}{3w-1}}$

Raychaudhuri Eqn VS Cosmological Const

R Eqn

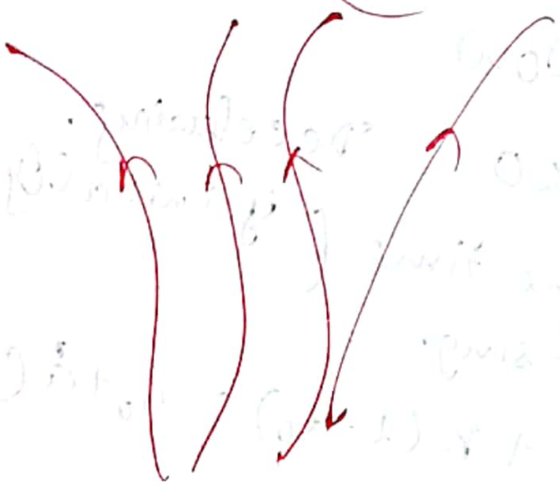
$$\frac{d\theta}{d\tau} = -\frac{\theta^2}{3} - \sigma^{\alpha\beta}\sigma_{\alpha\beta} + \omega^{\alpha\beta}\omega_{\alpha\beta} - R_{\mu\nu}U^{\mu}U^{\nu}$$

SEC : $R_{\mu\nu}U^{\mu}U^{\nu} > 0$

$$\therefore \frac{d\theta}{d\tau} < 0$$

(Rate of Expand. Universe will decrease)

$\theta > 0$



SEC: $r^2(\rho + a^2 p_1 + b^2 p_2 + c^2 p_3) \geq \frac{(\rho - p_1 - p_2 - p_3)}{2}$

$$T^{\alpha\beta} = \begin{pmatrix} \rho & & & \\ & -p_1 & & \\ & & -p_2 & \\ & & & -p_3 \end{pmatrix}$$

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(i) if $a=b=c=0$ $\quad v=0 \Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1$

$\rho + p_1 + p_2 + p_3 \geq 0$.

(ii) if $b=c=0$ $\quad \gamma = \frac{1}{\sqrt{1-a^2}}$

$$\boxed{\rho + p_i \geq 0}$$

(9) Exp.

$\dot{a} > 0$

$\dot{H} = \frac{\dot{a}^2 - \ddot{a}a}{a^2}$

$\dot{H} = H^2 - \frac{\ddot{a}}{a}$

$H^2 - \dot{H} = \frac{\ddot{a}}{a} > 0$

for exp^n $\boxed{H > 0}$

$\frac{\dot{H}}{H^2} < 0$ $\leftarrow \dot{H} - H^2 < 0$

$H_0^{-1} = T$ age of Universe

<u>Cosmological Const</u>

for ~~$p + 3\rho$~~ $\rho + 3p > 0$

Violate $\boxed{\begin{matrix}\rho + 3p > 0 \\ \text{SEC}\end{matrix}}$

to obtain age of universe $H_0^{-1} = T$ greater

$T^{\alpha\beta} = (p+\rho)\, U^\alpha U^\beta - p g^{\alpha\beta}$

for observer which are comoving with the exp^n. For all others $T^{\alpha\beta}$ would be different i.e. Different content of Energy/ Pressure.

But $\exists$ a case where $T^{\alpha\beta}$ same for every Observer regardless of U^α $\Rightarrow$ This occurs when $\boxed{p = -\rho}$

$T^{\alpha\beta} = -p g^{\alpha\beta} = \rho g^{\alpha\beta}$

Energy Consv^n $\quad \nabla_\alpha T^{\alpha\beta} = \rho \cancel{\nabla_\alpha g^{\alpha\beta}} + (\partial_\alpha \rho) g^{\alpha\beta} = 0$

$\Rightarrow \partial_\beta \rho = 0 \Rightarrow \rho = \underline{\text{const}}$

$$\therefore T_{\alpha\beta} = \rho\, g_{\alpha\beta}$$

$$T_{\alpha\beta} = \frac{\Lambda}{8\pi G} g_{\alpha\beta}$$

$$T_{\alpha\beta}(\Lambda) = \text{Vacuum Energy}$$

$$T_{\alpha\beta} = (\rho, -p, -p, -p)$$

$$T^{\alpha}_{\ \beta} = (\rho, p, p, p)$$

$$T^{\alpha}_{\ \beta} = \left(\frac{\Lambda}{8\pi G}, \frac{-\Lambda}{8\pi G}, \frac{-\Lambda}{8\pi G}, \frac{-\Lambda}{8\pi G}\right)$$

$$\rho = \frac{\Lambda}{8\pi G} \qquad p = -\rho$$

$$p = -\frac{\Lambda}{8\pi G} \qquad \text{or}$$

$$w = \frac{p}{\rho} = -1$$

Violates SEC

$$\boxed{\rho + 3p = \rho - 3\rho = -2\rho < 0}$$

$\therefore$ age of Universe $\nearrow$

$$\therefore \text{EFE} \Rightarrow G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$$

$$= 8\pi G T_{\alpha\beta} + \frac{8\pi G \Lambda}{8\pi G} g_{\alpha\beta}$$

$$\boxed{G_{\alpha\beta} - \Lambda g_{\alpha\beta} = 8\pi G T_{\alpha\beta}}$$

(16) Cosmological Observations (38)

$$M = -2.5 \log_{10} L + \text{const.}$$

Absolute Magnitude — const. & can be choosen arbitrarily depending on observed waveband.

(e.g. $M_{sun,B} = 5.48$ (B is Blue band at 4400A)).

(17) In Non expanding Euclidean Geom

$$f = \frac{L}{4\pi d^2}$$

$$m = -2.5 \log f + \text{const}$$

Apparent magnitude

[const choosen s.t. const = 0 for $f = 2.5 \times 10^{-5}$]

Friedman Eqn

$G_{00} - \Lambda g_{00} = 8\pi G T_{00}$

$H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho$

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$

Cty Eqn

$$m = -2.5 \log 2.5 \times 10^{-5}$$

$$M = -2.5 \log_{10} f 4\pi d^2$$

$$M = m - 2.5 \log_{10} 4\pi d^2$$

$$M = m - 5 \log_{10} d - 2.5 \log_{10} 4\pi$$

$$M$$

$$\boxed{H^2 = H_0^2 (\Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\gamma a^{-4} + \Omega_{0,\Lambda})}$$

$\frac{\Lambda}{3H_0^2} = \Omega_\Lambda$

d : Mpc

$$\boxed{m = M + 25 + 5 \log d}$$

$\mu = m - M$ = dist. Modulus & a measure of Distance.

ΛCDM model

① 3 components →

1. Λ = Dark energy
2. CDM → Non Rel. $p=0$ $\quad \rho \propto a^{-3}$
3. Ordinary matter ~~Relativist.~~

By FLRW

Ω_m

Ω_r

Ω_k

Ω_Λ

Quintissence vs cosmological constant

② ~~Dark energy vs Ordinary Matter~~

Baryons	$\rho \propto a^{-3}$	$p=0$
Radiation	$\rho \propto a^{-4}$	$p=\frac{\rho}{3}$
CDM	$\rho \propto a^{-3}$	$p=0$

Λ

$$H(a)=\frac{\dot a}{a}=H_0\sqrt{(\Omega_c+\Omega_b)a^{-3}+\Omega_\gamma a^{-4}+\Omega_k a^{-2}+\Omega_{DE}\, a^{-3(1+w)}}$$

③ ~~CDM~~ General

$p = w\rho \longrightarrow w=-1 \quad \Omega_{DE}=\Omega_V$

$\rho \propto a^{-3(w+1)}$

Now put in 1st F Eq to get $\boxed{a(t)}$ &

(4) FRW metric

(homogenity & isotropy)

&

(Spatial component is time dependent)

$$c^2 d\tau^2 = c^2 dt^2 - a^2(t)\, d\Sigma^2$$

Scale factor

3 dimensional space of Uniform Curvature. i.e. Elliptical space, Euclidean or Hyperbolic

$\Omega_m = \Omega_b + \Omega_c$

(4) Assuming $\Omega_k = 0$

& $\Omega_r << 1$

$$\therefore a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \sinh^{2/3}\left(\frac{t}{t_\Lambda}\right)$$

$$t_\Lambda = \frac{2}{3H_0\sqrt{\Omega_\Lambda}}$$

solving for $a(t) = 1$ we get t the age of the Universe

(8) Success & failure

As these observations show that our universe is made up of 2 components : Dark matter & Dark Energy.

Dark Matter; contributes 1/3 of total energy density of universe. ~25%

Dark Energy : Self interacting, negative pressure contributes ~~2/3~~ of total energy density of universe. 70%

$\Omega_\Lambda = 0.7$

The ~~matter~~ universe is made up of different components : matter, radiation & dark ~~energy~~ matter

Quintessence Model

This model also suffers from fine tuning problem. The problem is why does dark energy start dominating over matter content of universe recently.

... gas : Constraints from CMBR & SNIe

① The Universe is expanding was found out by the observations. One such observation was of SNe - Ia. There are many # of observations after this discovery of SNe were done to confirm expansion of Universe. These observations are Baryon Acoustic Oscillations (BAO), CMB, growth of structures & GRBs.

② There are many different cosmological models which explain the expanding Universe. One such is ΛCDM model (Λ cold Dark Matter) where is Λ is the cosmological constant. Accounts for the energy density of space or the vacuum energy. However this suffers from serious fine tuning problem. Hence, many models have been explored.

③ All models can be put into 2 classes.

Reforming geometry part of Einstein's Eq.

Reforming matter component of the Einstein Eq.

↓

In this exotic matter with negative pressure is added to the mass distribution of the Universe.

Expansion can't be explained if normal (observable) matter is taken into consideration.

↓

Some of the models of cosmo exp. based on this are: quintessence, k-essence, tachyons, Barotropic fluid etc. Collectively they are known as Dark Energy models.

Slowly evolving & spatially homogeneous Scalar field model or 2 coupled field (Quintisence, k-essence)

$P = f(\rho)$ (Barotropic fluid)

This also contains models ... Eq of State.

(5) Barotropic fluid $P = f(\rho)$.

Relation B/w P & ρ (energy density) determines the dynamics of the fluid. For eg. one such fluid is chaplygin gas.

mimics dark energy & dark matter & is a possible substn of stand. model of cosmo.

(8) Chaplygin Gas Model

Eq of state $\boxed{P = -\frac{A}{\rho}}$

(A is positive constant)

(p & ρ both in comoving frame with $\rho > 0$)

Generalized model of chaplygin gas

Eq of state (10) $P_{ch} = -\frac{A}{\rho_{ch}^{\alpha}}$ $A > 0$, $0 < \alpha \leq 1$

By energy Momentum Eq in flat FRW metric

$\boxed{d(\rho a^3) = -p\, da^3}$ (*)

we get $\rho_{ch} = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}$.

a is the scale factor

B is Integeration constant

Derivation: from (*)

$$a^3 d\rho + 3a^2\rho\, da = \frac{A}{\rho^\alpha} 3a^2 da$$

$$a\, d\rho = \left(\frac{A}{\rho^\alpha} - \rho\right) 3\, da$$

$$\frac{d\rho}{\frac{A}{\rho^\alpha} - \rho} = 3\frac{da}{a} \Rightarrow \frac{(\alpha+1)\rho^\alpha d\rho}{A - \rho^{\alpha+1}} = 3(\alpha+1)\frac{da}{a}$$

$$= \frac{d\rho^{\alpha+1}}{A - \rho^{\alpha+1}} = 3(\alpha+1)\frac{da}{a}$$

$$-\ln\left(A - \rho^{\alpha+1}\right) = \ln a^{3(\alpha+1)} c$$

$p = 0$ $\rho \propto a^{-3}$

$p = \frac{\rho}{3}$ $\rho \propto a^{-4}$

$$A - \rho^{\alpha+1} = a^{-(3\alpha+1)} c$$

$$\rho^{\alpha+1} = A - \frac{c}{a^{3(\alpha+1)}}$$

$$\boxed{\rho = \left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{\frac{1}{\alpha+1}}}$$

Doubts

① How BAO, CMB, growth of structure & GRB explain the expansion of the Universe.

② How these evolving & spatially homogenous scalar field ("Quintissence" model) works?

③ what is CDM model?

④ Difference B/w Dark matter & Dark Energy?

⑤ What is Energy momentum conservation statement in FLRW metric?

⑥ Why at Early time $\alpha = 1$

→ This is the fluid Eqn which holds for all radiation, matter & dark matter

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(P_i + \rho_i) = 0 \qquad i = \{r, b, ch\}$$

$r \downarrow$ rad, $b \downarrow$ Baryon matter, ch Chaplage

$$\frac{d\rho}{\partial t} + 3H(P + \rho) = 0$$

for Baryonic matter

$P = 0 \Rightarrow \dot{\rho}_b + 3\frac{\dot{a}}{a}\rho_b = 0 \Rightarrow d\ln\rho = -3\, d\ln a$

$\Rightarrow \rho = \rho_{b0}\, a^{-3}$

ρ_{b0} ↓ Int. Const.

for Radiation

$p = \frac{\rho}{3} \Rightarrow \dot{\rho}_r + 4\rho_r\frac{\dot{a}}{a} = 0 \Rightarrow \rho = \rho_{r0}\, a^{-4}$

ρ_{r0} ↓ Int const.

(6) $a << 1$ (Early times) $\xrightarrow[\rho=\frac{\sqrt{B}}{a^3}]{\alpha=1}$ $\rho \propto a^{-3}$ $\longrightarrow$ $\boxed{\text{CDM model}}$

Dust like matter or Dark matter

$a >> 1 \longrightarrow$ $8\pi G A^{1/1+\alpha}$ or $p = -\rho = \text{const}$ $\rightarrow$ Λ model

(7) ~~CMB~~ Generalized Chaplygin gas model is succesfull in SNe test, CMB peak locations, Gravi lensing etc.

However it produces oscillations or exponential blowups of matter power spectrum which is inconsistent with observation. $\therefore$ Modification of Gen. chaplygin gas

$\downarrow$

Variable chaplygin gas

(8) Variable chaplygin Gas

Eq of state $\quad P_{ch} = -\frac{A(a)}{\rho_{ch}}$

$$A(a) = A_0 a^{-n}$$

A_0, n is constant
$A(a)$ is a positive fn.
a is scale factor.

Using energy moment conservation in flat FRW metric we get

$$\rho_{ch} = \sqrt{\frac{6}{6-n}\frac{A_0}{a^n} + \frac{B}{a^6}}$$

Derivation:

$\rho = A^{\frac{1}{\alpha+1}} = \text{const}^{n} = \frac{A}{\rho^{\alpha}}$

$p = -\frac{A}{A^{\frac{\alpha}{\alpha+1}}} \Rightarrow p = -A^{\frac{1}{\alpha+1}}$

$= -\rho$

$\therefore \; p = -\rho = \text{const.}$

Transformation from $z \to a$

$1 + z = \frac{a_0}{a}$ where a_0 is the scale factor at present time which we normalize to 1.

$\boxed{\text{Condition on } n\text{?}}$

(3) $a \ll 1 \longrightarrow a^{-3}$

$a \gg 1 \longrightarrow p = -\rho = \text{const.}$

Chaplygin gas evolves from dust dominated epoch to cosmological const. in present times

Derivation:

if $n < 6$

$$\left(\frac{6}{6-n}\frac{A_0}{a^n}\right)^{1/2}$$

$\rho \quad a^{-n/2}$

(10) Defining $\Omega_m = \dfrac{B}{\frac{6A_0}{6-n} + B}$

we get $\rho_{ch}(a) = \rho_{ch,0}\left(\dfrac{\Omega_m}{a^6} + \dfrac{1-\Omega_m}{a^n}\right)^{1/2}$

$$\rho_{ch,0} = \sqrt{\frac{6}{6-n}A_0 + B}$$

↑ present value of ρ_{ch} → $a = 1$ A_0

(11) Friedman Eqn in terms of $\Omega_m, \Omega_b, \Omega_\gamma$

$$H^2 = \frac{8\pi G}{3}\left\{\rho_{\gamma 0}(1+z)^4 + \rho_{b0}(1+z)^3 + \rho_{ch0}\left[\Omega_m(1+z)^6 + (1-\Omega_m)(1+z)^n\right]^{1/2}\right\}$$

$$H^2 = \Omega_{ch,0} H_0^2 a^{-4} X^2(a)$$

$$X^2(a) = \frac{\Omega_{\gamma 0}}{1-\Omega_{\gamma 0}-\Omega_{b0}} + \frac{\Omega_{b0}\, a}{1-\Omega_{\gamma 0}-\Omega_{b0}} + a^4\left(\frac{\Omega_m}{a^6} + \frac{1-\Omega_m}{a^n}\right)^{1/2}$$

Doubts

① Relation b/w a & z?
② How to find condition on n?

$$H^2 = H_0^2 \left[\Omega_\gamma (1+z)^4 + \Omega_b (1+z)^3 + \Omega_{cho} [\ ] \right]^{1/2}$$
$$= H_0^2 \Omega_{cho} (1+z)^4 \left[\Omega_\gamma + \frac{\Omega_b}{1+z} + \frac{\Omega_{cho}}{(1+z)^4} [\ ] \right]^{1/2}$$

⑫ **Luminosity Distance**

$$d_L = \frac{c}{a H_0} \int_{a_{es}}^{1} \frac{da}{\Omega_{cho}^{1/2} x(a)} \qquad dz = -\frac{da}{a^2}$$

or

$$d_L = c(1+z) \int_0^z \frac{dz'}{H(z', p)}$$

$$\mu = 5 \log \frac{H_0 d_L}{ch} + 42.38$$

$$h = \frac{H_0}{100 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}}$$

---

① Consider a galaxy of physical size 4 kpc. Calculate the angle subtended by this galaxy assuming it is situated at redshift 0.7. Calculate this for Euc. Universe where universe is matter dominated.

② Find how the energy of massive non-relativistic particle changes as Universe expands. [redacted] $\text{flux} = \frac{1 L_\odot}{\mathrm{Mpc}^3}$

⑧ SN is a std. Candle $L = 4\times10^9 L_\odot$ (Peak Lum) Angular size of Galaxy can be found if we know Angular Size of Galaxy using SN as std candle.
① Size of Galaxy
② Find diameter of Galaxy in terms of its Ang. size

8

Statistical Analysis

① $\Omega_{b0} = 0.02$ $\qquad \Omega_{b0} = \frac{\rho_{b0}}{3H_0^2}$

$\Omega_{r0} = 0.0000245$ $\qquad \Omega_{r0} = \frac{\rho_{r0}}{3H_0^2}$

② $\frac{10^{\mu/5+1}}{10^6} = d_L$ in Mpc

③ To determine the best fit parameters, we minimize

$$\chi^2 = \sum_i \left[\frac{\mu_{th}^i - \mu_{obs}}{\sigma_i}\right]^2 - \frac{C_1}{C_2}\left(C_1 + \frac{2}{5}\ln 10\right) - 2\ln h$$

$$C_1 = \sum_i \frac{\mu_{th}^i - \mu_{obs}^i}{\sigma_i^2}$$

$$C_2 = \sum_i \frac{1}{\sigma_i^2}$$

~~Sudipta Sarkar~~ 25 ~~BH course~~

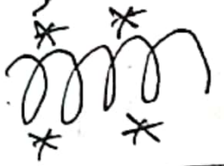
~~L-1~~

(1)

~~(10) Key Properties of Gravitational Waves~~

Just like we calculate Angular size of the moon.

also if the star is moving then we will get due to proper motion of star



we can learn about dist + vel. perpendicular of the star

we can measure till 500pc away stars by parallax by Hipparcos satellite

Gaia Satellite measures till 30000ly. → we can get dist.

$F = \frac{L}{4\pi r^2}$

from this we can get luminosity

By putting filters through light from these stars we can get color of these stars.

Lum vs Color

HR diagram → Definite pattern which is used for finding

1. Stellar evolution
2. distance

Calibration

HR Diagram as Std Candle

Method

Main Sequence-fitting

let say star is too far for parallax to find dist

Measure the color & ~~luminosity~~

we can know the range of luminosity through HR diag.

∴ Range of distance

$F = \frac{L}{4\pi r^2}$

There would not be one stars but clusters of them (globular cluster)

1. measure color of each
2. ~~either~~ guess dist. & then measure through L
3. Match it with HR diagram
4. choose which fits HR diagram

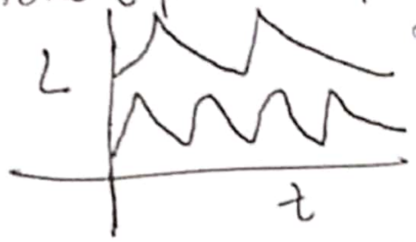
→ works till 10^5 pc

covers Most of Milky Way

Can't cover whole Universe Because for longer range color of stars can't be distinguish

Cosmic Distance ladder: Cepheid Variables

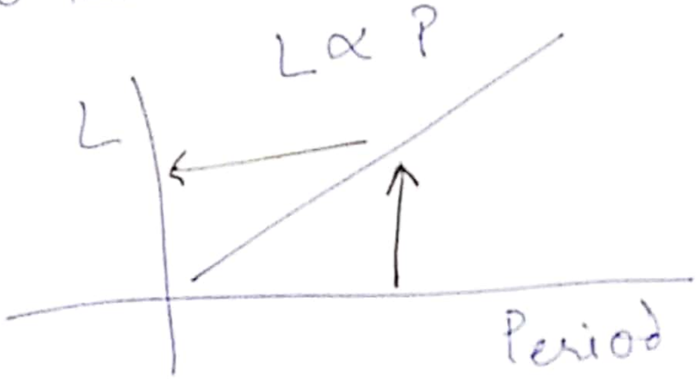
1. Stars usually evolve over long time scale i.e. they are constant forus. But there is a type of star (variable star) which changes very quickly.
2. One subtype of variable star is cephaid variables which can be used to measure distance.
3. Period is related luminosity. Long period Cepheid variable have higher luminosity than short period Cepheid variables.



2 stars with same distance ∴ Luminosity is not changing due to distance but it is intrinsic property

There is a relationship B/w L & period

If we know the period of cepheid then we can know the lum. thus the distance.



But farther than few 10 MLy we can't measure Bec. star is not that bright ∴ we need other std. Candles which are bright

4. This method was applied to V1 in M31 galaxy which is closest galaxy to Milky way by Hubble. This proved that V1 was out side our galaxy & Our Milky way is not the only galaxy.

was at 2.35 MLy away

34 Each ... on earlier rung measurement ← MMA technical ① → ∴ errors carry through.

Cosmic Distance Ladder: Std. Candles

①

$$F = \frac{L}{4\pi r^2}$$

we know luminosity intrinsically

flux is how many photons are hitting.

∴ we get r.

① L from HR diagram. ← Sp HR diagram fitting

from observable property $\boxed{\text{color}}$ & then relate it to L & then find r

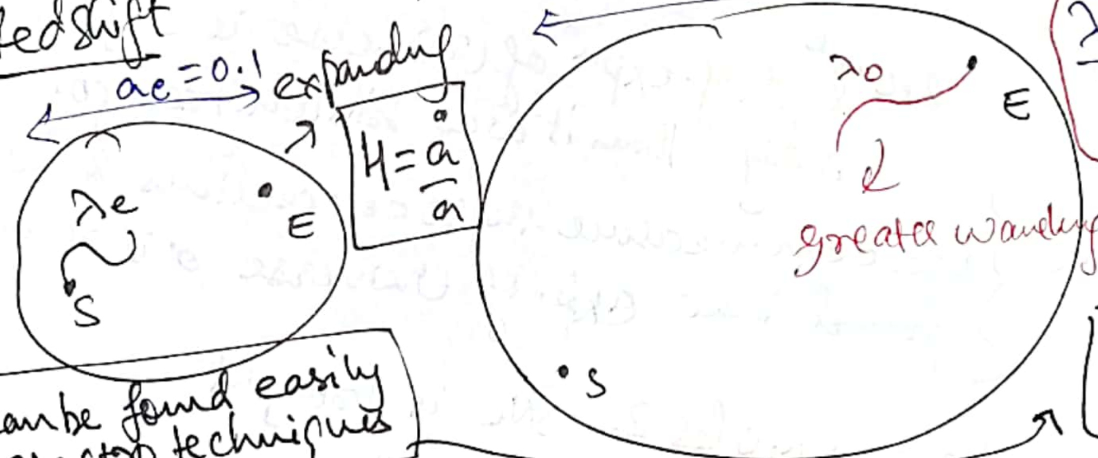
→ Std. Candles

② Redshift

a = scale factor ∴ universe exp 10 times.

$a_e = 0.1$ expanding $\boxed{H = \frac{\dot{a}}{a}}$ $a_0 = 1$

$$\boxed{\frac{\lambda_0}{\lambda_e} = \frac{a_0}{a_e}}$$



$\boxed{\lambda_e \text{ can be found easily by spectro techniques}}$

$\boxed{\text{each element has unique spectrograph}}$

$$\text{Redshift} = z = \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{v}{c}$$ → recession of source w.r.t. observer (Assuming $v << c$)

for nearby source $v = H_0 d$ (ignoring vel. of galaxy through gravity)

$$\therefore z \approx \frac{H_0 d}{c}$$

∴ If I know redshift of galaxy, I know H_0 const. ∴ I can get distance

$H_0 = 70.4 \pm 1.4$ km/s/Mpc

if source is 1 Mpc ($10^6 \times 3.26$ lt yr) away then that source is moving at 70.4 km/s away

14 (32)

③ ∴ How to measure red shift

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e}$$

By spectral lines

This redshift method can measure distance upto > 200 MLy

Errors

① In lower redshift, galaxies have their own intrinsic motion in addition to the Universe being expanding.
∴ this adds error while finding distance
as $v = H_0 d$ is no longer valid

② At higher redshifts i.e. early time of the Universe $v = H_0 d$ is not valid
in fact $v = H_0 d$ + corrections

Because Rate of expⁿ of Universe is different today than it was billion yrs ago.

We can measure these corrections & we found that exp. of Universe is accelerating

SN measurements helped in finding these corrections & helped to find universe is accelerating

④ SNe

Star explodes & SNe is brought

There are different types of SNe and all types show variability
i.e. the graph of lum. is not consistent every time except of type IA SNe.

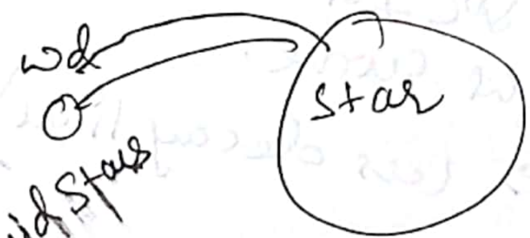
(33)

3) Type Ia mechanism: Not core collapse

$< 1.4 M_\odot$ can't do SNe bec. they are small & are stable due to e^- deg. pressure

orbiting binary

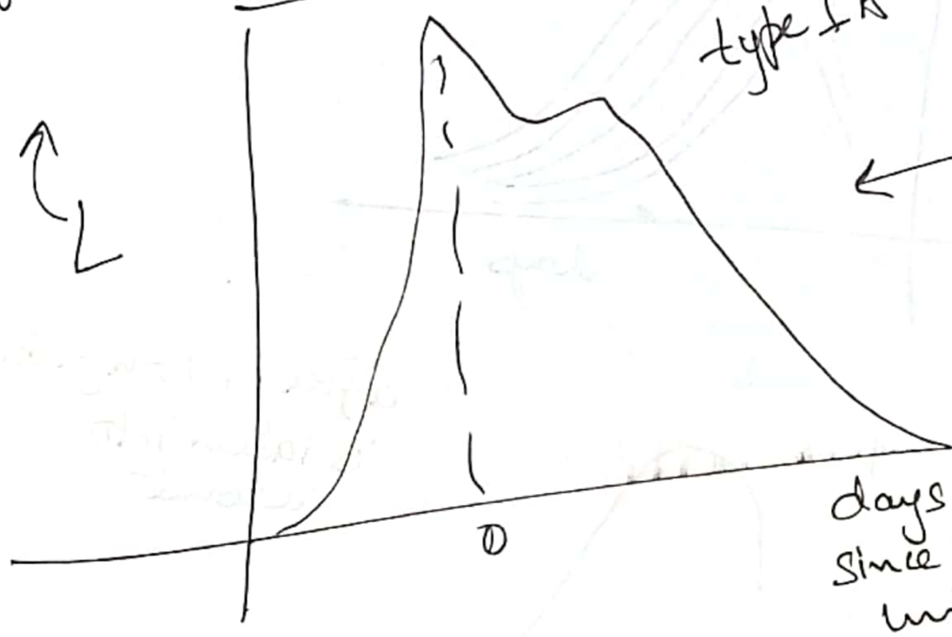
→ outer layer of star gets accreted into wd. ∴ ↑ mass of wd ∴ Carbon fuse & outside press & SN



Assume same galaxy as of cepheid, this SN happened ∴ we know dist. we also know flux ∴ L

Calibrate it through cepheid stars

Stable Std. Candle



As SN type IA occurs by the same mechanism the L–t graph should be more or less same
~~∴ we know period~~
∴ we can measure distance

Now we know that SNe are standard candles

4) Now we know that SNe are "Std candles" i.e. L is same
if 2 sources have same intrinsic lum ("Std candles")
∴ from the ratio of their apparent Brightness we can derive the ratio of their ratio of luminosity dist.

(34)

(7) This is in analogy with

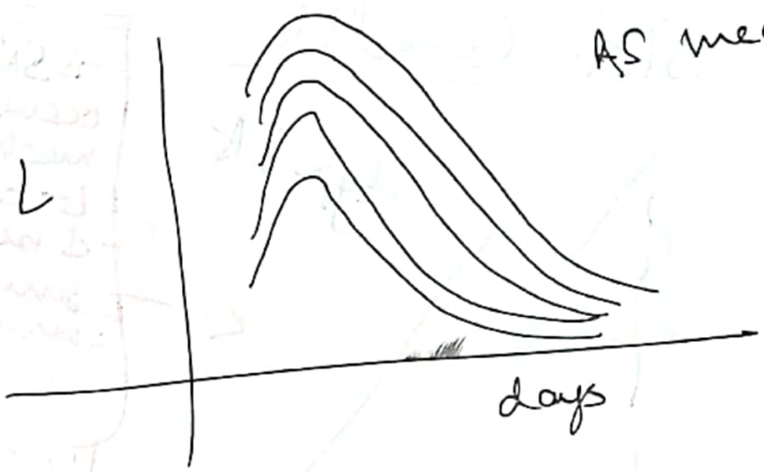
"If we have 2 sources of same physical size then from the ratio of their apparent angular size
We can derive ratio of angular diameter distance

(8) Now if SNe physics is complex & can't be Modelled ∴ Why SNe is a Std Candle.

Reason;

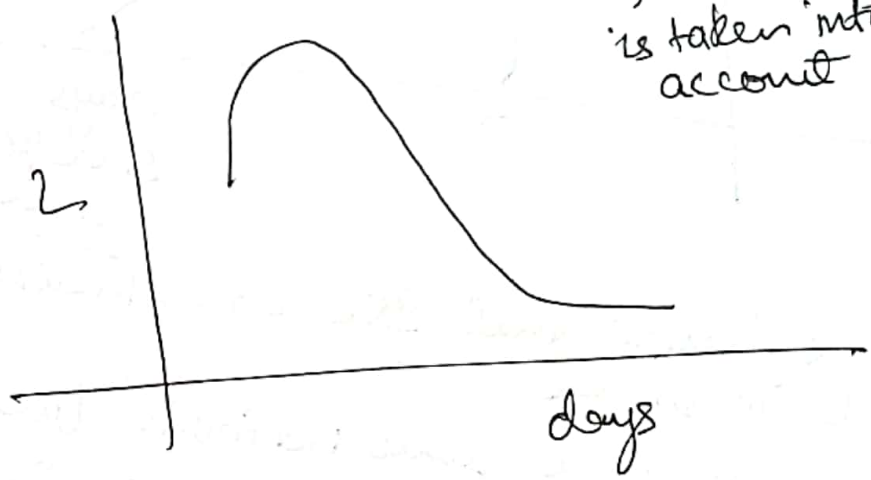
Peak Brightness of SNe Ia correlates with the shape of its light curve.
(More the peak ⇒ less decay time)

As measured



Luminosity peak is at $M_V \sim -19.3$

after stretch factor is taken into account



∴ They all follow same light curve shape

∴ Taking into stretch factor we can standardize the ~~candle~~ SNe. to 10 %. or better

However, the absolute zero-point of SNe a distance scale has to be caliberated externally eg. with cephids. But cephids are not many which has SNe also.

4 ~~We can~~

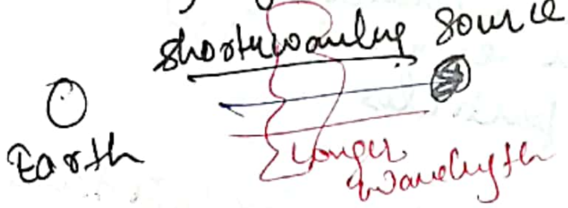
Cosmic distance ladder cons.

1 Some object (earlier in the ladder) may not be present in the same galaxy of the other object which we are trying to use as std. candle or later in cosmic distance ladder.

2 Interaction of light with IS dust. which makes light dim which can make us think source is farther

We can account for 2 by

let say if we measure ligh



we know Interstellar dust absorbs longer λ but not short λ

∴ We can find that it would be due to IS dust

But if all λ are changed then due to distance.

We can study properties of IS dust.

3 errors in earlier ladder can keep on adding in later ladder

Because we use in later ladder the earlier ladder

(10) We can also standardize SNe not only by Stretch factor but also by color information.

Supernova Cosmology with Python

(1)
- SN cosmology
  - describe scientific problem
  - describe the data to set up the issues
- Python Code SNCosmo & features
- Use of simulation catalogs being built for LSST survey.

"The Universe is an excellent Lab for testing Physics"

→ Can Probe physical effects at a very long time Scales

→ Probes effects at large spatial scales

(2) Dark matter : Observational evidence for gravity Stronger than expected for observed particles

Dark Energy : Observational evidence for late time acc. of universe.

(3) SN cosmology was the first evidence that universe is expanding.

These people involved in this were given Nobel Prize.

④ How can we get information on cosmic expansion from data (37) 17

- Combine knowledge of astrophysical systems, the impact of the expansion & observations of those system.
- for SN : This means comparing observed brightness which is affected by expansion of the universe to intrinsic brightness.

Data

⑤

Scanned with CamScanner

Daniel Holz
Scott Hughes
Bernard Schutz.

Measuring Cosmic Distances with Standard Sirens

(1) Because GW encodes the distance to its source, GW170817 provided astrophysical community with another advance: the first measurement of the local cosmic rate: Hubble constant - via GW.
This is the new ~~...~~ way for standard siren technique.

(2) If light emitted from source has wavelength λ & is at distance D from observer.

By def $\frac{\lambda_{obs}-\lambda_{em}}{\lambda_{em}} = z$ $\quad \lambda_{obs} = (z+1)\lambda_{em}$

Then
Observer will measure the light to have a wavelength of $(1+z)\lambda_{em}$ where z is the redshift.

(3) To leading order in z, (for nearby objects)
$$cz = H_0 D \rightarrow \text{Physical Distance}$$

H_0 is today's value of Hubble parameter H.
Dimensions s^{-1} $\quad$ $km\ s^{-1}\ Mpc^{-1}$
But astronomers measure in units
because Mpc is suitable for IG distances.
$Mpc = 3.26$ million lgt yrs. $= 3.26 \times 10^6$ ly

(4) $\frac{1}{H_0} = [s] \equiv$ Hubble time $\equiv$ Age of the Universe

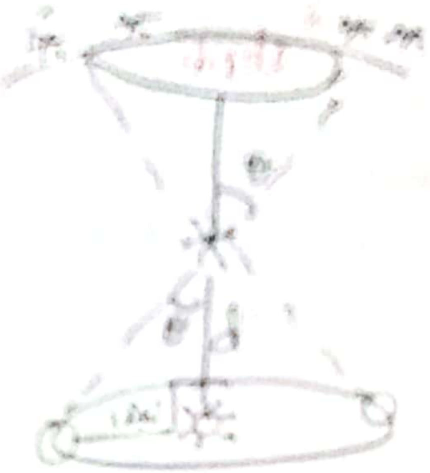
(5) for far away objects) higher order terms needs to be added

(6) Redshifts can be determined from spectral measurements but determining astronomical distances is difficult

⑦ For Nearby Objects

Distance through Parallax

How to measure θ
(Tilting of Telescope!)



If the star is not at 90° to then then it will be circular but if 0° then it would be line.

Assuming back ground stars are far away that parallax doesn't matter = ignore

$$d = \frac{1\,AU}{\tan\theta}$$

$$d(pc) = \frac{1}{\theta(\text{arc sec})}$$

$1\,pc = 3.26\,ly$

$\text{arc sec} = \frac{1}{3600}°$

We can measure stars closer to us by this method as θ becomes too small to measure for large dist stars

The technique doesn't work for large distances, as angular shift due to Earth's orbital motion becomes too small to measure.

⑧ **Standard Candle**: Astronomical source whose intrinsic luminosity is assumed to be known.

Cosmic Distance Ladder: There are multiple methods for measuring distances. Objects thought to be of std. candle is identified on each ring & calibrated in terms of measured to previous rung.

⑨ SN are also std candles & they helped determine H_0.
They also implied non linear contribution to
$cz = H_0 D$
implying that universe is accelerating.

(10) Key Properties of GW

→ $h_{\mu\nu}$ is analogous to A_μ of EM

j: spatial

$$A_j = \frac{\mu_0}{4\pi}\frac{1}{D^2}\frac{dp_j}{dt}$$

(H) for source moving at less than speed of light

$\vec{p}$ = Electric Dipole moment.

D = Distance from source.

μ_0 = Permeability of free space

$$p = \int_{\text{source}} \rho_e r \, dV$$

ρ_e: Charge Density

volume of the source

→ Analogous result for GW

$$h_{jk} = \frac{2G}{c^4}\frac{1}{D}\frac{d^2 I_{jk}}{dt^2}$$

j, k: spatial as GW are ortho to propogation dir

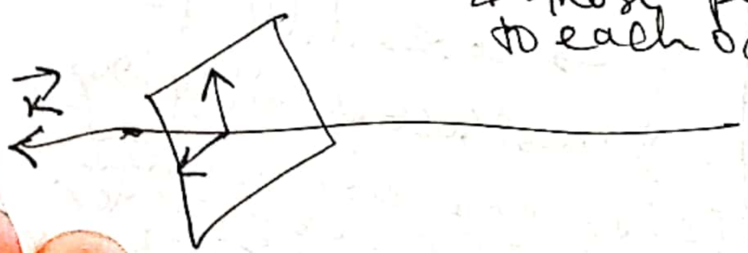
Amplitude falls off $\propto \frac{1}{D}$

I_{jk}: Quadropole moment

$$I_{jk} = \int_{\text{source}} \rho_m \left[r_j r_k - \frac{r^2}{3}\delta_{jk} \right] dV$$

(11) Polarization of GW

In EM, polarizations are in plane $\perp^r$ to propog. & those polarizations are orthogonal to each other



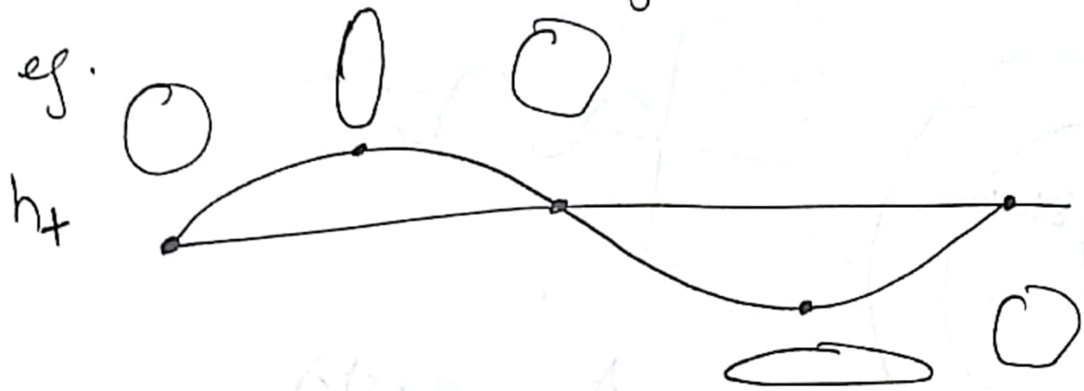
m: GW, polarization Basis are In plane ⊥ to propagn
But they are rotated by 45° to eachother.

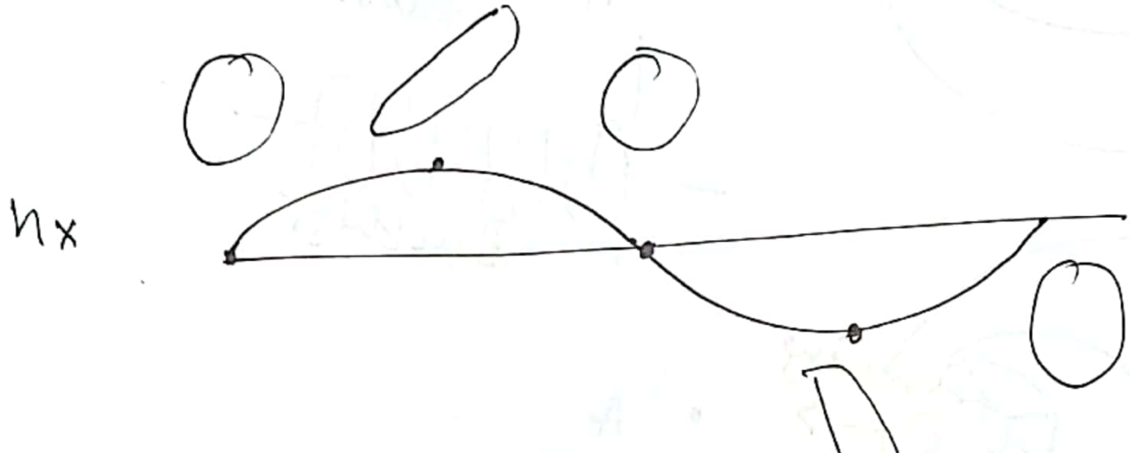
(42)

GW stretches & squeezes along pol. basis

EM force exerts force on charge along polarization basis.

eg.





(12) If somehow we know how source quadrupole moment varies with time, then we can measure distance.

↓

for case of Binary inspiral quad. moment (t) can be known & hence distance. ∴ Without any reference to cosmic distance ladder, distance can be measured of Binary inspirals.

(13) From kepler's law & formula which relates $\frac{dI}{dt}$ to I_{jk}

$$\frac{d\Omega}{dt} = \frac{96}{5}\left(\frac{GM}{c^3}\right)^{5/3}\Omega^{11/3};$$

Ω: frequency of orbiting

M: Chirp mass

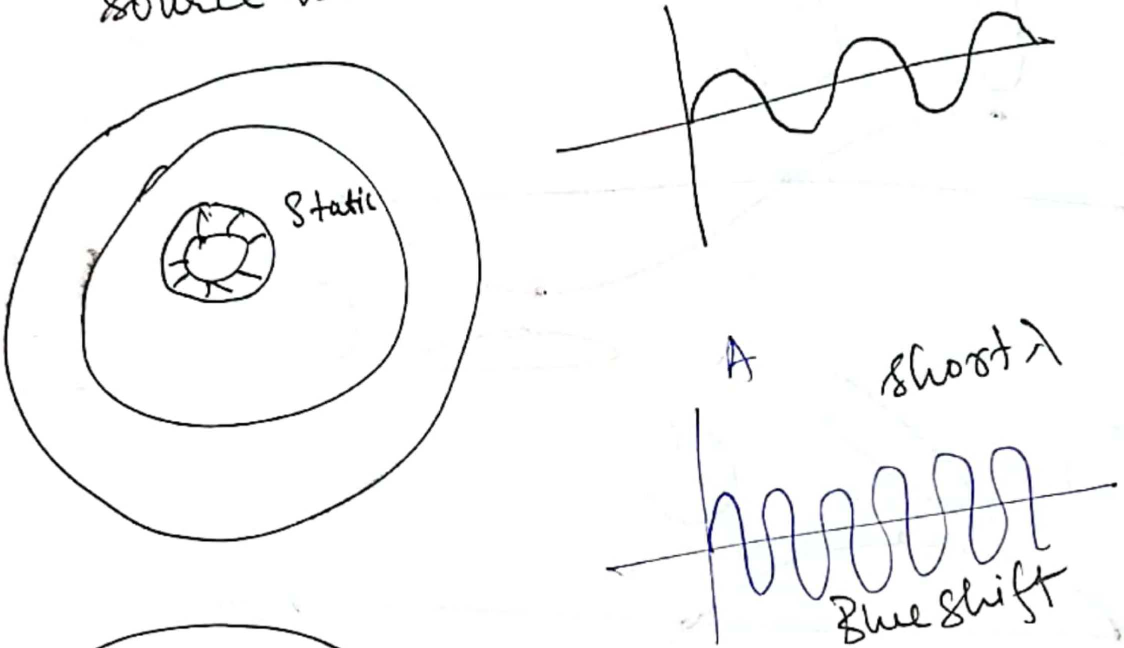
$$M = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

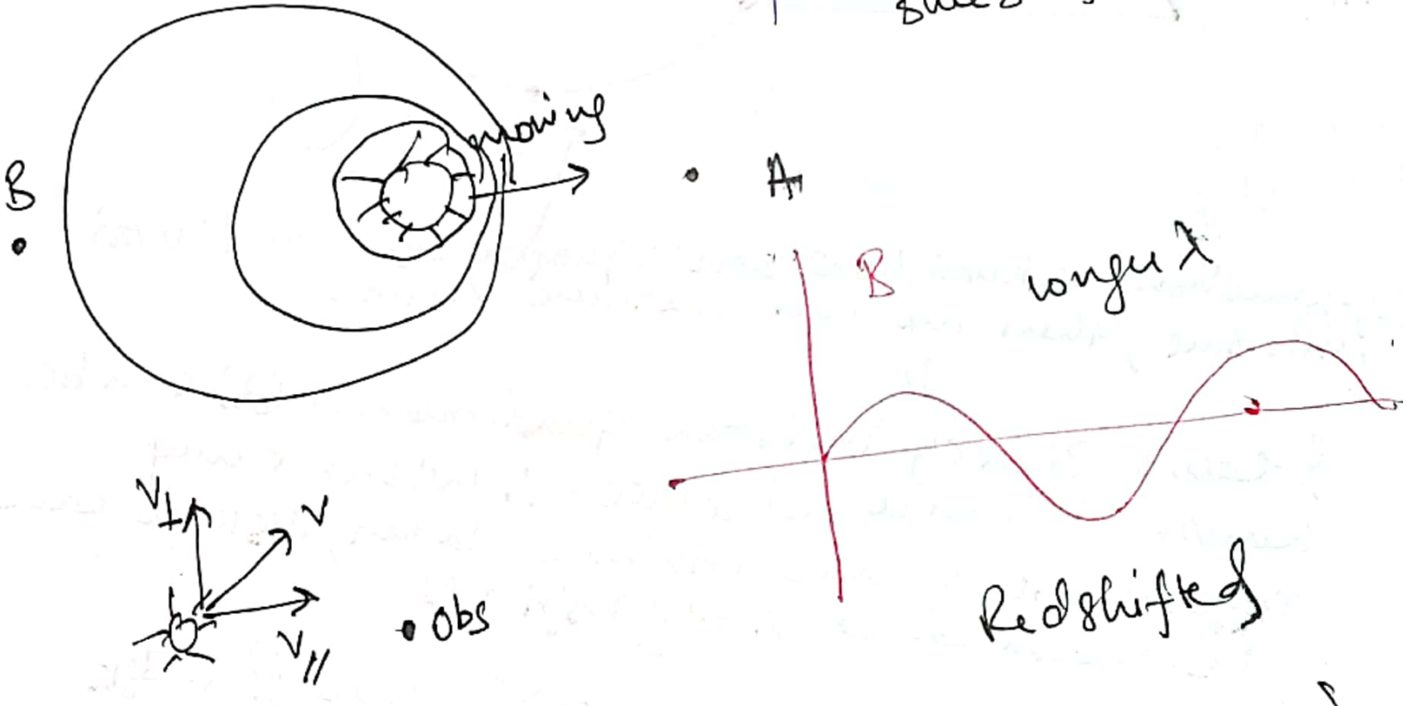
to the leading order

Once we know M, we know change in freq.
Same M, Same $\Omega(t)$ even though m_1 & m_2 of 2 system differ.

~~Redshift & Hubble's Law~~
Doppler effect

① frequency & wavelength depends on the source movement.





$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{emit}} \approx \frac{v_\parallel}{c} \quad \left(\text{assuming source } v << c\right)$$

2 λ_{em} can be found through spectrograph.

44 if there are multiple lines at spectrograph & each shifted by same amount then we know which line corresponds to which & thus λ_{emit} can be known.

if only one line is there then we can figure if the shifted line we observe is of the same element

~~Doppler & SB~~

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(16) Cosmological Observations

$$M = -2.5 \log_{10} L + \text{const.}$$

Absolute Magnitude

const. & can be choosen arbitrarily depending on observed waveband.

(e.g. $M_{sun,B} = 5.48$ (B is Blue band at 4400A)).

(17) In Non expanding Euclidean Geom

$$f = \frac{L}{4\pi d^2}$$

$$m = -2.5 \log f + \text{const}$$

Apparent magnitude

[const choosen s.t. const = 0 for $f = 2.5 \times 10^{-5}$]

$$m = -2.5 \log 2.5 \times 10^{-5}$$

$$M = -2.5 \log_{10} f\, 4\pi d^2$$

$$M = m - 2.5 \log_{10} 4\pi d^2$$

$$M = m - 5 \log_{10} d - 2.5 \log_{10} 4\pi$$

$$M$$

$$\boxed{m = M + 25 + 5 \log d}$$

d : Mpc

$\mu = m - M$ = dist. Modulus & a measure of Distance.

Freidman Eqn

$G_{00} - \Lambda g_{00} = 8\pi G T_{00}$

$H^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi G}{3} \rho$

$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$

Cly Eqn

$$\boxed{H^2 = H_0^2 (\Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_r a^{-4} + \Omega_{0,\Lambda})}$$

$\frac{\Lambda}{3H_0^2} = \Omega_\Lambda$

(18) Absolute Magnitude (56)

Apparent Magnitude

Given any cosmologic model (Ω_m, Ω_Λ etc)
we can obtain
(1) $r(z)$
(2) $d_L(z)$
(3) $\mu \xrightarrow[\text{M}]{\text{knowing}}$ (4) m

Relativistic version

(19) $$f = \frac{L}{4\pi r^2 (1+z)^2}$$

$$d_L = r(1+z) \quad \therefore f = \frac{L}{4\pi d_L^2}$$

$$\therefore m - M = \mu = 5 \log d_L + 25.$$

(1+z) appears BCZ. (1) Energy emitted is redshifted away. (2) Time interval of
dt_0 recieved $= \frac{a_0}{a_1} dt_1 \rightarrow$ time of emission

(20) Coordinate distance r along null geod. i.e

$$ds^2 = c\,dt^2 - a^2 dr^2 = 0$$

(i) $k=0$ $$\Rightarrow r = \int_0^r dr' = c\int_{t_1}^{t_0} \frac{dt}{a} = c\int_{a_1}^{a_0} \frac{da}{\dot{a}a} = c\int_{a_1}^{a_0} \frac{da}{Ha^2}$$

$$dz = -\frac{da}{a^2} \Rightarrow r = c\int_0^z \frac{dz}{H(z)}$$

(ii) Non flat $$r = \frac{1}{H_0\sqrt{|\Omega_k|}} S\left[\sqrt{|\Omega_k|}\int_0^z \frac{dz'}{E(z')}\right]$$

$\boxed{H = H_0 E(z)}$

$$\boxed{\Omega_k = -\frac{k}{H_0^2}}$$

$$S(x) = \begin{cases} \sin(x) & k=+1 \\ x & k=0 \\ \sinh(x) & k=-1 \end{cases}$$

GW & Cosmology

Daniel Holz

① Overview → (i) Cosmological Backgnd
(ii) GW
(iii) Std. Sirens → (i) SMBH
(ii) GRBs

(iv) Grav. Lensing.

② Cosmology → We are trying to measure evolution history of the Universe.
i.e. we are trying to get Lum. Dist. – Redshift Curve.

Lum. Dist → tells how much time light took to reach ∴ tells abt time. $t = \frac{d_L}{c}$

Redshift → tells abt size of Universe

∴ LD – Redshift tells about size as fn of time of Universe.

③ Redshift → Scale Ratio of universe at time of emission

$$a(z) = \frac{a_0}{1+z}$$

Measuring Redshift is straight forward.

④ Distance → Measuring distance luminosity is the hardest & we use std sirens for that.

(5) (10) (47) <u>Std. Sirens</u>: <u>SNe</u> → we know intrinsically how bright they are & hence by luminosity we can tell how far they are.

SNe → is good to about 10 - 15%.

∴ The motivation is if we can do good more than 10-15%

(6) To know the physics behind $d_L - z$ of SNe is difficult we have to know (1) Element abundance of star
(2) Convection
(3) Neutrinos.

∴ The best simulations are fairly poor

(7) On the other hand, BH are described by just 3 no.
BH binary requires 15 #.
∴ They are std. objects ∴ They can be modelled extremely well.
<u>GW std</u>

(8) Strongest harmonic:

$$h(t) = \frac{M_z^{5/3} f(t)^{2/3}}{D_L} F(\text{angles}) \cos\phi(t)$$ (Post Newtonian form

Dimensionless strain $h(t)$
luminosity distance D_L
accumulated GW phase $\cos[\phi(t)]$
GW frequency $f(t) = \frac{1}{2\pi}\frac{d\phi}{dt}$
position & orientation dependence $F(\text{angles})$ → depends on the inclination of observer
Redshifted Chirp Mass

$$M_z = (1+z)\frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

11. By GW templates we can measure M_z uniquely.
. Other things also can be taken care of except D_L
. So we know $h(t)$, M_z
we can infer D_L

This is an absolute measure of D_L
in case of SNe D_L is relative measure of distance. (Schutz 1986)

⑨ Detecting GW

GW are weak.
fractional strain due to strong GW $h \sim 10^{-22}$
This much distance (m) we are trying to measure

further there is Noise

⑩ Redshift is intrinsically built in chirp mass
$\therefore$ M_z has 3 parameters z, m_1, m_2
$\therefore$ GW templates we can measure M_z but
$\therefore$ we can't tell redshift.
So SMBH & stellar BH can have similar templates depending on the redshift of Both. $\because$ gravitationics scale free

⑪ In Astronomy $\rightarrow$ Easier to detect z
difficult to detect d_L

In GW $\longrightarrow$ Easier to detect d_L
difficult to detect z.

⑫ GW provide direct measure of luminosity distance, but they give no independent information about redshift.

(13) GW is scale free

- GW signal from local binary with masses (m_1, m_2) is indistinguishable from binary with masses

$\frac{m_1}{1+z}$ $\frac{m_2}{1+z}$ at redshift z.

∴ To measure cosmology, we need independent determination of redshift.

∴ EM counterparts do help in determining redshifts.

(14) The other way to get redshift is by the statistics.

i.e. Let say we know the population of SMBH masses we can know z

or

if we know masses of NS individually we can infer z.

(15) We can measure very accurately d_L z ..

But Grav. Lensing brings errors.

Motion due to expn of Univ	+	Motion due to other forces eg. gravity

~~Dopp ... & S R~~

Hypothesis Testing

Basics

① Hypothesis: Something we can put to a test.

> So, the question we are interested in is: let say, we took some sample, & then we put our Hypothesis to test against this sample. & then ask

How much does the sample data put doubt on our Hypothesis?

Doubt

> let say, sample isn't putting any doubt against hypothesis but is it still safe to say our Hypothesis is correct by just checking one small sample size.

→ Type 2 errors

② Doubt against our Hypothesis can be made in 2 ways

① The result of the sample is far away from our H_0

② The result is far away but the sample size is small (i.e. less doubt on H_0)

③ Critical values: values beyond which we reject our H_0

Critical values depend on the sample size:

for eg for small sample ⇒ $H_0: \mu = 20$

↓

$\mu = 25$

c ← 8 → 8 → c — critical value.

12 — 20 — 28

Reject

for large sample size → allow H_0 can be true.

$(\mu = 22)$

3 ← 3 → 23

c — 20 — c

(1) (4) Question: If H_0 is true, how extreme is our sample?

① if sample mean is too far away from hyp. mean then prob. H_0 is fal[se]

② if sample is too big & mean is sti[ll] far it makes the case worse than ①

formula: $z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

if $z = 0$ or $\bar{X} = \mu$ then H_0 True

if $z \neq 0$ & $n \uparrow\uparrow$ then H_0 false.

(5) Type 1 errors: Occurs when you reject a null hypothesis that is infact true.

Type 2 errors: Occurs when you accept a null hypo that is infact false.

There is still a possibility that H_0 True prob $= \alpha$

α ↓ level of Significa[nce]

We can choose level of significance.
Larger the level of signif. less the critical values.
(More strict we were)

Convention
5% level of Sig[nificance]

There is a possibility that H_0 is false Prob. $= \beta$

$1 - \beta =$ Power of Hyp. Test

21/11/21 54

Meeting

1. What should be the σ value in case of GW as errors in it ~~are~~ have different up & low limit?
2. How to make contours & what is the meaning of confidence intervals in your graphs?
3. Can we also use Chisquare in case of d_L?
4. Why Chisquare for GW is so low??
5. → Transformation $y = \sqrt{2\chi^2} - \sqrt{2\,dof - 1}$

   should follow Gaussian

   $y < 1 \Rightarrow$ within 1σ of mean 68.3 % prob of correct fit

   $y = 3 \Rightarrow$ 0.6 % prob. of correctly fit

$$\frac{\text{Chi square sum}}{\text{No. of data sets}} \approx 1 \quad \text{for good fit}$$

Chi Square minimum for diff. parameter will give most prob. value of the parameter which fits well

6. How to take care of asymmetric errors in curve-fit?

53 events

~~Who calculated~~