# Tests of General Relativity and Black hole spectroscopy with quasinormal modes

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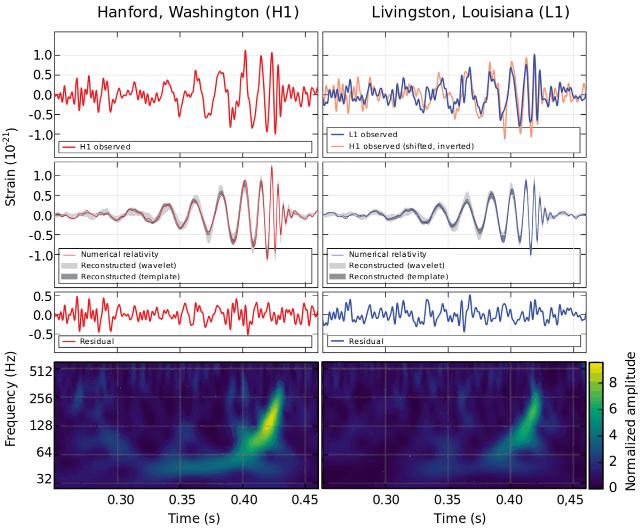
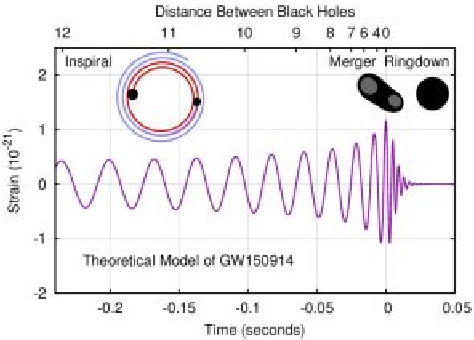
# The first detection - GW150914



# Inspiral-Merger-Ringdown




- Ringdown phase is well described by perturbing the background black hole spacetime by any test field.
- It is assumed that this model is valid only when black hole is very close to reaching its equilibrium.

# Motivation

- **Stability of black holes**: Given the initial perturbation, will the perturbation remain bounded at all times?

  [Regge, Tullio and Wheeler, John A. (*Phys. Rev* 1957]

- **Gravitational wave Astronomy**: To extract information of the black hole like their mass, charge, and spin.
- **Tests of General Relativity**: Different modes should give us a stringent test for GR. Using different techniques within GR independently to test GR.
- **Confirmation of BHs**: Detection of these frequencies also confirm the existence of the BHs.

# The aim of the project


What will happen to the black hole when it is perturbed by an external field?

- A perturbed black hole will go through quasinormal modes ringdown. The aim of the project was to calculate these quasinormal modes.
- As a toy model I will treat scalar fields as an external test field.

# Scalar Perturbation-Introduction

- Klein Gordan Equation for massless scalar field in spherically symmetric and static background

$$\Box\phi = 0$$

- Decomposing the function as

$$\phi(t, r, \theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{\Psi_{\ell m}^{s=0}(r)}{r} P_{\ell m}(\theta) e^{-i\omega t} e^{im\phi}$$

- By using tortoise coordinate $r_*$ defined as $\frac{dr}{dr_*} \equiv (fh)^{1/2}$, we get the Schrodinger equation:

$$\frac{d^2\Psi_l^{s=0}}{dr_*^2} + \left[\omega^2 - V_0\right] \Psi_l^{s=0} = 0$$

where effective potential reads as $V_0 \equiv f\frac{\ell(\ell+1)}{r^2} + \frac{(fh)'}{2r}$

# Boundary Conditions

- Effective Potential $V_0^{\text{Schw}} = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)$
- **Horizon**: Classically, nothing should leave horizon, therefore

$$\phi \sim e^{-i\omega(t+r_+)}$$

- **Infinity**: Discarding unphysical waves entering from infinity.

$$\phi \sim e^{-i\omega(t-r_+)}$$

- Now our aim is to solve for $\omega$ in the radial equation satisfying the above boundary conditions and show that the eigen functions are damped in nature.

# WKB Method

- For any massless spin (s=0 scalar, s=1 Electromagnetic, s=2 Gravitational):

$$\frac{d^2\Psi_\ell^s}{dr_*^2} + Q(r_*)\Psi_\ell^s = 0$$

  where $Q(r_*) = \left[\omega^2 - V_s\right]$, $V_s = f\left(\frac{\Lambda}{r^2} + \frac{2\beta}{r^3}\right)$, $\beta = 1 - s^2$, $\lambda = \ell(\ell+1)$

- **Validity**: Due to boundary conditions, the transmitted and reflected waves should have comparable amplitudes. Therefore, turning points degenerate or very close to each other.
- In middle region, expanding Q about maxima and using boundary conditions, we get

$$n + \frac{1}{2} = -\frac{\mathrm{i}Q_0}{\left(2Q_0''\right)^{1/2}}$$

- Using the above result to get

$$\omega_n^2 = i\left(n + \frac{1}{2}\right)\sqrt{2Q_0''} + \left(1 - \frac{2M}{r_0}\right)\left(\frac{\Lambda}{r_0^2} + \frac{2\beta M}{r_0^3}\right)$$

# Application

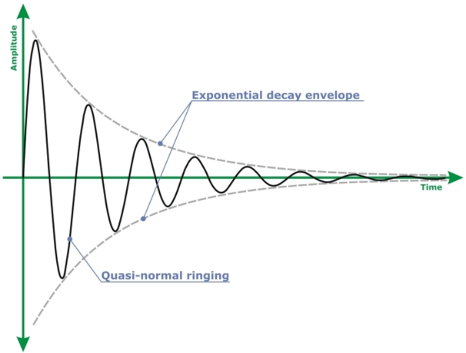
$M = nM_\odot$

**For gravitational field, s=2**

| $l$ | $n$ | $M\omega_0 + iM\omega_i$ |
|---|---|---|
| 2 | 0 | $0.373162 - 0.0892174i$ |
| | 1 | $0.346017 - 0.274915i$ |
| | 2 | $0.302935 - 0.471064i$ |
| | 3 | $0.247462 - 0.672898i$ |
| 3 | 0 | $0.599265 - 0.0927284i$ |
| | 1 | $0.582355 - 0.281406i$ |
| | 2 | $0.5532 - 0.476684i$ |
| | 3 | $0.515747 - 0.677429i$ |
| 4 | 0 | $0.809098 - 0.0941711i$ |
| | 1 | $0.796499 - 0.284366i$ |
| | 2 | $0.773636 - 0.478974i$ |
| | 3 | $0.743312 - 0.6783i$ |

**For scalar field in RN spacetime assuming** $q = 0.5$

| $l$ | $n$ | $Q$ | $M\omega_0 + iM\omega_i$ |
|---|---|---|---|
| 0 | 0 | 0.5 | $0.247376 - 0.249018i$ |
| | 1 | 0.5 | $0.887109 - 0.922351i$ |
| | 2 | 0.5 | $1.37073 - 1.41889i$ |
| | 3 | 0.5 | $0.142375 - 0.863119i$ |
| 1 | 0 | 0.5 | $0.26604 - 0.145384i$ |
| | 1 | 0.5 | $0.700671 - 0.66721i$ |
| | 2 | 0.5 | $0.992813 - 0.97177i$ |
| | 3 | 0.5 | $0.0604634 - 0.758635i$ |



# Inferences

- Converting into Physical units
  $\nu = \frac{32.26}{n}(M\omega_0)\,\mathrm{kHz}, \qquad \tau = \frac{n \cdot 0.4937 \cdot 10^{-5}}{(M\omega_i)}\,\mathrm{s}$
- Bandwidth of LIGO is 10 Hz - few kHz, Corresponding mass range of BH is

$$10M_\odot \lesssim M \lesssim 10^3 M_\odot$$

- Bandwidth of LISA is $10^{-4} - 10^{-1} Hz$ , Corresponding mass range of BH is

$$1.2 \cdot 10^5 M_\odot \lesssim M \lesssim 1.2 \cdot 10^8 M_\odot$$

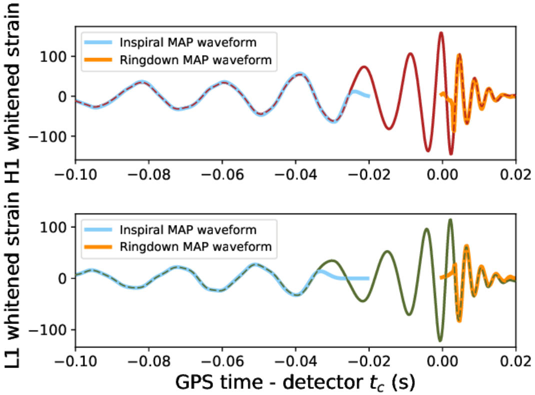
- Test no-hair theorem and stability of black holes
- In eikonal limit, we can relate QNM to lyapunov exponent

$$\omega_{QNM} = \Omega_c l + i\left(n + \frac{1}{2}\right)|\lambda|$$

  where $\lambda$ is the Lyapunov exponent (measures of the rate at which the trajectories diverge) of the null orbit.
  $\Omega_c$ is the orbital angular velocity of the null orbit.

# Open Question

- Can we quantify the beginning of the ringdown through observations?



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# Thank You

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