Variable Chaplygin Gas:
Constraints imposed from Gravitational Merger Events

Ashley Chraya
Advisor: Dr. Geetanjali Sethi

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Overview

FLRW Metric

- Assuming the universe is homogeneous and isotropic, we obtain,

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{1}$$

  where k= curvature of the space can take values $k = 0, \pm 1$ and a(t) is the scale factor.
- By taking time component of the energy-momentum conservation equation $T^{\mu\nu}_{,\mu} = 0$, we obtain the continuity equation,

$$\frac{\partial \rho}{\partial t} + 3H(p+\rho) = 0 \tag{2}$$

  where H is the Hubble parameter, $H = \frac{\dot{a}}{a}$, ρ - total density of Universe
  Note: Continuity equation remains same even after considering any dark energy model $p = w\rho$
- Using Einstein field equations, $G_{\alpha\beta} = 8\pi G T_{\alpha\beta}$, and FLRW metric, we obtain Friedmann equations

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) \end{aligned} \tag{3}$$

  Note1: If $T_{\alpha\beta}$ is changed, Friedmann equations are changed.
  Note2: The Friedmann equations and the conservation equations are however not independent.

Different Fluid Components

- Assuming, critical density $\rho_c = \frac{3H^2}{8\pi G}$ and density parameter $\Omega = \frac{\rho}{\rho_c}$, first Friedmann equation reads as

$$\Omega_b(a) + \Omega_k(a) = 1 \tag{4}$$

  This relation extends directly to other models with several components.

- **Non Relativistic fluid:** Such a fluid approximates “dust” matter (like e.g. galaxies) or non-interacting particles with non-relativistic velocities (like e.g. cold dark matter (CDM)).

$$p = 0 \tag{5}$$

- By continuity equation, we obtain the relation between matter density and scale factor $\rho \sim a^{-3}$
- **Relativistic fluid:**

$$p = \frac{\rho}{3} \tag{6}$$

  Note: Trace of Energy-Momentum tensor vanishes which is a general feature of any conformal invariant theory like EM theory.

- By continuity equation, $\rho \sim a^{-4}$
- **Exotic fluid:**

$$p = w\rho \tag{7}$$

- By continuity equation we obtain $\rho \sim a^{-3(w+1)}$

- Taking baryonic matter and radiation into account, Friedmann equation reads as,

$$H^2 = H_0^2(\Omega_{m,0}a^{-3} + \Omega_{\gamma,0}a^{-4} + \Omega_{k,0}a^{-2}) \tag{8}$$

where $\Omega_{m,0} + \Omega_{\gamma,0} + \Omega_{k,0} = 1$

Every other component can be added when its behaviour with scale factor is known.

- There are tons of models but who will decide from what model to choose?

Observations can put constraints on our theory!!

- Supernova observations suggest universe is expanding.

Expansion of Universe: A quick timeline

- 1912 - Vesto Slipher: Redshift of Galaxies
- 1923 - Alexander Friedmann: Expansion of Universe

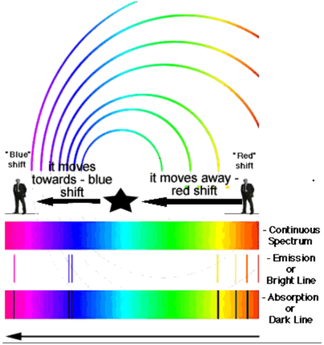

Figure: Redshift

Expansion of Universe: A quick timeline

- 1927 - George Lemaitre: Linear relation - Distance and Expansion of Universe.
- 1929 - Edwin Hubble: Observational proof for Lemaitre's hypothesis.

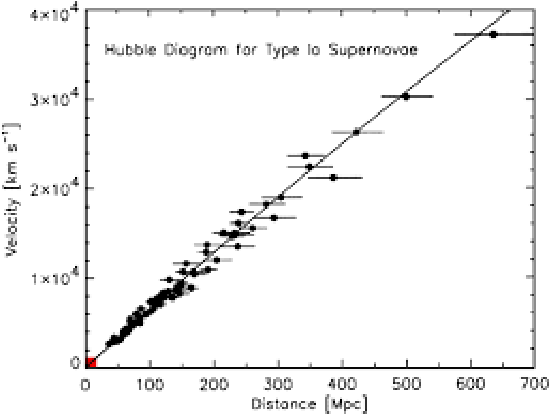

Figure: Hubble Graph

Inferences from Theory

- **Strong Energy conditions:** $R_{\alpha\beta}k^\alpha k^\beta \geq 0$ which for perfect fluid reads as
$$\rho + 3p \geq 0 \tag{9}$$
- If strong energy condition holds then from 2nd Friedmann equation, acceleration is negative.
- **Frobenius Theorem**: Congruence of Curves is Hypersurface Orthogonal iff
$$\nabla_{[i}U_j U_{k]} = 0$$
For Null geodesics, rotation parameter should vanish $\omega_{ab} = 0$
- **Raychaudhuri's Equation**: The evolution equation for the expansion scalar:
$$\frac{d\theta}{d\tau} = -\frac{\theta^2}{2} + \sigma^{\alpha\beta}\sigma_{\alpha\beta} - \omega^{\alpha\beta}\omega_{\alpha\beta} - R_{ij}k^i k^j$$
using $\tilde{B_{\alpha\beta}}\tilde{B^{\alpha\beta}} = B_{\alpha\beta}B_{\alpha\beta}$
- **Focusing Theorem**: For matter following Strong /Weak Null energy condition $R_{\alpha\beta}k^\alpha k^\beta \geq 0$ and the geodesic congruence be hypersurface orthogonal, the expansion must decrease during its' evolution.
$$\frac{d\theta}{d\lambda} \leq 0$$
For $\theta = \theta_0 < 0$ under these conditions θ goes to $-\infty$ along the geodesic within the affine parameter $\lambda \leq \frac{2}{\theta_0}$
The congruence will develop a caustic within finite proper time.
- Therefore, for expanding universe strong energy condition has to be violated.

ΛCDM Model

- There are many way to violate strong energy conditions. One way is to assume a constant being added to the gravitational Lagrangian which has the energy momentum tensor $T_{\alpha\beta} = diag(\frac{\Lambda}{8\pi G}, \frac{\Lambda}{8\pi G}, \frac{\Lambda}{8\pi G}, \frac{\Lambda}{8\pi G})$
- Therefore, equation of state for this exotic matter (dark energy) is

$$p = -\rho \tag{10}$$

or $w = -1$ in $p = w\rho$ equation of state.

- We can easily see it violates strong energy condition, therefore, expansion of the universe.
- Modified Friedmann equations

$$\begin{aligned} H^2 &= H_0^2(\Omega_{m,0}a^{-3} + \Omega_{\gamma,0}a^{-4} + \Omega_{k,0}a^{-2} + \Omega_{\Lambda,0}) \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \end{aligned} \tag{11}$$

- Λ **CDM:** Assuming baryonic matter, cold dark matter, radiation, curvature and dark energy component to exist, Friedmann equation reads as,

$$H^2 = H_0^2((\Omega_{b,0} + \Omega_{c,0})a^{-3} + \Omega_{\gamma,0}a^{-4} + \Omega_{k,0}a^{-2} + \Omega_{DE,0}a^{-3(1+w)}) \tag{12}$$

- These expansion can be directly correlated to the fact that the Universe was smaller earlier then what it is today. So, the idea of Big Bang began to develop.

Observational Motivations for Λ CDM model

- In 1930s, it was found that mass of the galaxies has to be higher than the visible matter to validate the higher orbital speed of the periphery of the galaxies. He proposed a presence of electromagnetically ”dark matter” causing such anomalous high speed of rotations.
- In early 1990s, with the observations of type Ia Supernovae (SNeIa), the expansion of the universe was found to be accelerated.
- With accelerated expansion, the universe needed additional presence of mass or energy to be explained with the current cosmological model.
- With no evidence on these additional matter from Electromagnetic Radiations, this additional energy termed as Dark Energy.

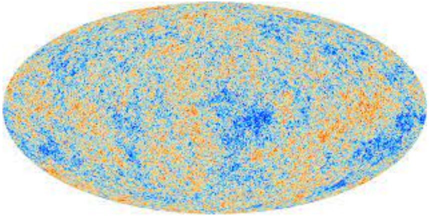

Figure: Cosmic Microwave Background

Problems in ΛCDM Model

- **Fine Tuning Problem:** The parameters like Critical Density bring a new problem - Is Universe fine tuned?
- **Universe is expanding!** - ”Why now”, that is, why does the dark energy start dominating over the matter content of the Universe recently?

Variable Chaplygin Gas (VCG)

- As an alternative to both the cosmological constant and cold dark matter, it is also possible to explain the acceleration of the universe by introducing a cosmic fluid component with an exotic equation of state, called Chaplygin gas.
- The attractive feature of such models are that they can explain both dark energy and dark matter with a single component. Equation of state of Chaplygin gas

$$p = \frac{-A}{\rho} \tag{13}$$

  where A is the positive constant.
- By continuity equation we can find the relation between Chaplygin gas density and scale factor.

$$\rho = [A \frac{B}{\alpha^3(\alpha+1)}]^{\frac{1}{\alpha+1}} \tag{14}$$

- We can easily see Chaplygin gas behaves like a non relativistic matter at early times while at late times the equation of state is dominated by a cosmological constant.
- Chaplygin gas model produces oscillations or exponential blowup of matter power spectrum that are inconsistent with observations. Therefore, variable Chaplygin gas model was proposed whose equation of state is

$$P_{ch} = -\frac{A(a)}{\rho_{ch}} \tag{15}$$

Variable Chaplygin Gas

- Friedmann equation of the variable Chaplygin gas model can written as

$$H^2 = \frac{8\pi G}{3}\left[\rho_{r0}(1+z)^4 + \rho_{b0}(1+z)^3 + \rho_{ch0}\left[\Omega_m(1+z)^6 + (1-\Omega_m)(1+z)^n\right]^{1/2}\right] \qquad (16)$$

where $\Omega_m \equiv \frac{B}{6A_0/(6-n)+B}$

Special Features of Variable Chaplygin Gas Model

- Account for Supersymmetry (Brane Models (d) brane in a (d+2) brane).
- The variable Equation of State.
- Incorporation of Dark Matter and Dark Energy as a single entity.

Essential Equations - VCG v/s ΛCDM

Distance Modulus ΛCDM Model,

$$d_L = a(t_0)r(1+z) \tag{17}$$

where a is the scale factor.
The r is coordinate distance and is expressed as

$$r = \int_t^{t_0} \frac{cdt}{a(t)} = \frac{c}{a_0 H_0} \int_0^z \frac{dz'}{h(z')} \tag{18}$$

$$h(z) = [\Big((1-\Omega_{total})(1+z)^2 + \Omega_m(1+z)^3 + \Omega_\Lambda(1+z)^p\Big]^{1/2} \tag{19}$$

in a flat universe, Ω_{total} = 1 and p is numerically equal to 0. So, the equation becomes

$$h(z) = [\Omega_m(1+z)^3 + \Omega_\Lambda\Big]^{1/2} \tag{20}$$

therefore,

$$d_L = \frac{1}{H_0^2(1+z)} \frac{c}{a_0} \int_0^z \frac{dz'}{[\Omega_m(1+z')^3 + \Omega_\Lambda]^{1/2}} \tag{21}$$

$$\mu_{obs} = 5 \log d_L - 5 \tag{22}$$

Essential Equations - VCG v/s ΛCDM

Distance Modulus *VCGModel*,

$$d_L(z,\mathbf{p}) = c(1+z)\int_0^z \frac{dz'}{H(z',\mathbf{p})} \tag{23}$$

where z is the redshift.

H is given by ,

$$H^2 = \frac{8\pi G}{3}(\rho_{r0}(1+z)^4 + \rho_{b0}(1+z)^3 + \rho_{ch0}[\Omega_m(1+z)^6 + (1-\Omega_m)(1+z)^n]^{1/2}) \tag{24}$$

$$\mu_{th} = 5\log d_L(z) - 5\log h + 42.38 = 5\log d_L(z) + 5log\left(\frac{cH_0}{1Mpc}\right) + 25 \tag{25}$$

Cosmic Distance Ladder

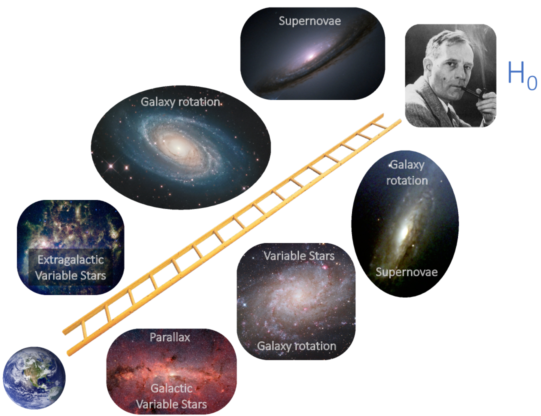

Figure: Distance Ladder

Standard Sirens and Standard Candles

Why Type 1a Supernova (SNe1a) are termed as Standard Candles?

Peak Luminosity - Obtained from Carbon-Oxygen White Dwarf beyond Critical Mass (1.44 Solar Mass) - Brighter Entity to measure Redshift (In other words, energy from the source isn't lost much)

Gravitation Wave from Merger Events - Standard Siren!

They interact the least with both Baryonic Matter and the Exotic entities like Dark Matter or Dark Energy - no energy lost - therefore reliable

Cosmology from Gravitational Waves

- Quadrupole formula for gravitational waves:

$$h_{jk} = \frac{2G}{c^4}\frac{1}{D}\frac{d^2 I_{jk}}{dt^2} \tag{26}$$

where j,k are purely spatial, and I_{jk} is the source's mass quadrupole moment.

- Gravitational waves generated by binary stars as an independent cosmic distance ladder!
-

$$\begin{aligned} h_+ &= \frac{2c}{D}\left(\frac{GM}{c^3}\right)^{5/3}\Omega^{2/3}\left(1+\cos^2 i\right)\cos 2\Phi(t) \\ h_\times &= \frac{4c}{D}\left(\frac{GM}{c^3}\right)^{5/3}\Omega^{2/3}\cos i \sin 2\Phi(t) \end{aligned} \tag{27}$$

where chirp mass $M = (m_1 m_2)^{3/5}(m_1 + m_2)^{-1/5}$, i is the inclination angle and can be determined if more than one polarization is measured, Φ is the accumulated orbital phase found by integrating the orbital frequency Ω over the duration t of the measurement.

$$\frac{d\Omega}{dt} = \frac{96}{5}\left(\frac{GM}{c^3}\right)^{5/3}\Omega^{11/3} \tag{28}$$

Project Overview

- The GWOSC (Gravitational Wave Open Science Center) has the dataset of all the observed gravitational merger events present in them. The data of the merger events include Luminosity Distance, Value of Inspiral Masses, Chirp Mass of the binary system, Time of the Event and much more.
- Vital information for us needed among those data - Redshift and Luminosity Distance

Two standard siren approaches

- EM counterpart/Bright - Gravitational waves provide distance and photons provide redshift. For example- GW170817

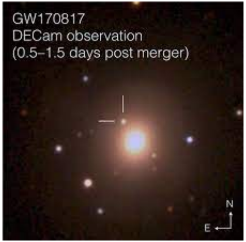

Figure: Unique host galaxy

- Statistical/Dark - If you can't identify the unique host galaxy, then use all galaxies in the 3D localization volume

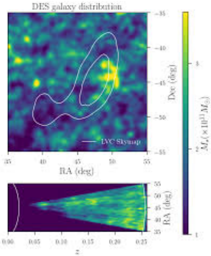

Figure: Use all galaxies in localization volume

The Crux of the Project

- Distance Modulus obtained from Luminosity Distance is used to find the best fit for the Distance Modulus obtained from the Variable Chaplygin Gas Model as a function of redshift.
- The parameter Ω_m and n are parameters of the VCG Model are constrained.

Methodology

- Distance Modulus obtained from Luminosity Distance
- Distance Modulus obtained from the Variable Chaplygin Gas Model as a function of redshift (μ_{th})
- χ^2 test between μ_{obs} and μ_{th} to obtain the χ^2 goodness value

Results

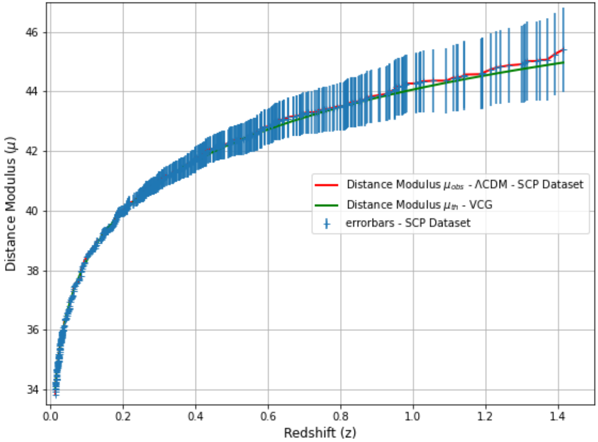

Figure: Performance of VCG Model to determine Distance Modulus (Green) with respect to the Distance Modulus from SCP Dataset (Red) against Redshift - Ω_m=0.15, n = 0.79 and H_0=69.8

Applications

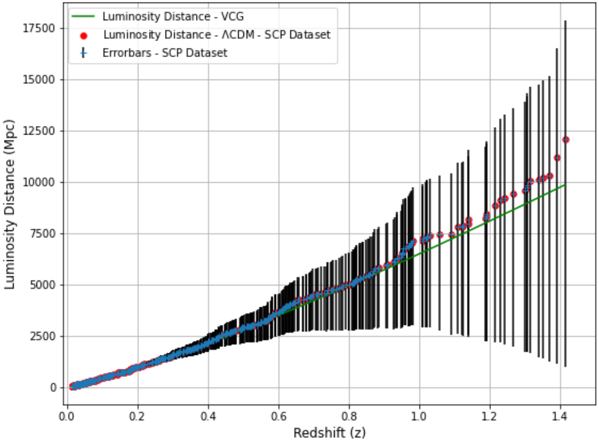

Figure: Performance of VCG Model to determine Luminosity Distance (Green) with respect to the Luminosity Distance from ΛCDM Model (Red) against Redshift - Ω_m=0.15, n = 0.79 and H_0=69.8

Applications

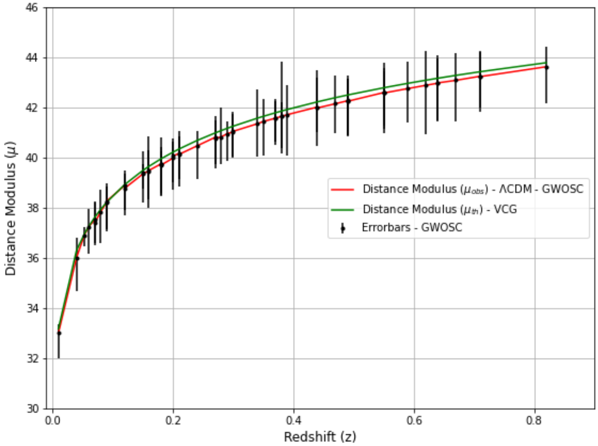

Figure: Performance of VCG Model to determine Distance Modulus (Green) with respect to the Distance Modulus from ΛCDM Model (Red) against Redshift from GWOSC Dataset

Applications

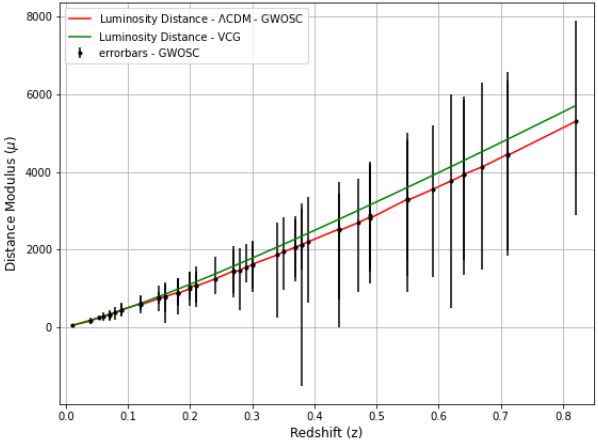

Figure: Performance of VCG Model to determine Luminosity Distance (Green) with respect to the Luminosity Distance from ΛCDM Model (Red) against Redshift from GWOSC Dataset

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Thank You